Chapter 9

9.1 When homogeneous isotropic turbulence decays it enters a low Reynolds number 'final period' where the longitudinal correlation coefficient function is known to be given exactly by the formula

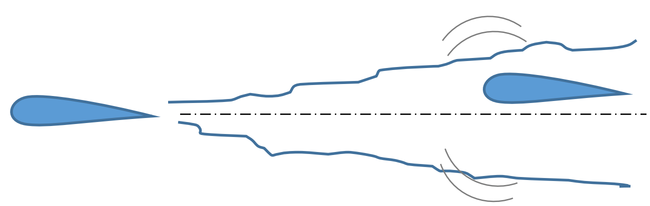
$$f(r) = e^{-r^2/8\nu t}$$

where t is time and ν is the kinematic viscosity (Batchelor, G. K. and Townsend, A. A. (1948). "Decay of turbulence in the final period." Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences **194**(1039): 527-543). Denoting the mean square velocity fluctuation by $\overline{u^2}$ where needed, determine,

(a) The longitudinal integral scale of the turbulence L_f
(b) The lateral correlation function $g(r)$.
(c) The one-dimensional wavenumber spectrum $\phi_{11}(k_1)$ (analogous to equation 9.1.16).
(d) Plot on a log-log scale the one-dimensional wavenumber spectrum, normalized by $\overline{u^2}L_f$, as a function of $k_1 L_f$ from 0.01 to 100 and compare with that implied by a von Kármán spectrum. Use a vertical axis from 1e-4 to 1. Comment on whether a Gaussian correlation function is likely to provide an accurate description of turbulence in high Reynolds number applications.

The turbulence is being swept along by a uniform flow of velocity U in the x_1 direction. Making, and stating any needed assumptions, determine expressions for

(e) The spectral density of streamwise velocity fluctuations seen at a fixed point $S_{11}(\omega)$
(f) The spectral density of streamwise velocity fluctuations seen at a fixed point $G_{11}(fL_f/U)$



9.2. Consider sound being generated by a pair of parallel airfoils, one placed downstream of the other. The second airfoil is positioned so that it cuts the wake of the first airfoil along a plane one half-wake width above the wake center plane. In order to calculate the unsteady loading on this second airfoil we need to know the space time correlation function of the upwash velocity fluctuations in this cutting plane. The goal of this question is to plot this function. Go to https://aeroacoustics.net. Locate the two-point wake data. Download and study the data file structure and sample codes. Write a code to evaluate and contour the desired correlation function as a function of spanwise separation Δx_3 from -$2L_w$ to $2L_w$ and for time delays from $-2L_w/U_\infty$ to $-2L_w/U_\infty$. Normalize your plot on the mean square upwash velocity at this point.

Worked example solution

Solution Problem 9.2

```
clear all; close all;

load('ruiuj.mat');
y0=1;
dz0=-2:.1:2;
dy0=0;
dt0=-2:.1:2;
[dz,dy,y,dt]=ndgrid(dz0,dy0,y0,dt0);
R22=squeeze(corrn(ruiuj,2,2,y,dy,dz,dt));

contourf(squeeze(dz),squeeze(dt),R22/max(R22(:)),20);
xlabel('\Deltax_3/L_w');ylabel('\tauU_\infty/L_w');
colorbar;
axis image;
```