Chapter 14

14.1 It is important to understand that at low Mach number the noise of a leading edge in turbulence is produced by disturbances that have a supersonic or nearly supersonic trace speed spanwise across the leading edge. Investigate the reasoning behind this statement using equations 14.2.1 and 13.4.5 as starting points and considering whether the parameter κ is real or imaginary. Contrast the different decay rates of disturbances downstream of the leading edge in these two cases. Show explicitly that the real κ implies a supersonic or nearly supersonic trace speed. Explain why the trace speed can sometimes be subsonic. Note that 14.2.1 is miss-printed in the book and should appear as:

$$\tilde{p}(\mathbf{x},\omega) \approx -\frac{i\pi\omega x_2 e^{ik_o r_e - ik_o M x_1}}{c_\infty r_e^2}\Delta\tilde{\tilde{p}}(k_1^{(o)}, k_3^{(o)}, \omega) \quad \text{where}$$

$$k_1^{(o)} = k_o\left(\frac{x_1}{r_e} - M\right) \text{ and } k_3^{(o)} = \frac{k_o x_3 \beta^2}{r_e}$$

Worked example solution

14.2 Write a Matlab code to calculate and compare the far field leading edge noise spectrum for a flat plate with a central 10cm of span subject to a homogeneous turbulent flow with three different characteristic lengthscales. Use equation 14.3.5 to compute the spectral density of the far field noise given the conditions below. Choose appropriate values for all other required parameters assuming the airfoil is in air. Assume the observer is directly above the center of the airfoil (chordwise and spanwise) 1m from its surface. Submit your code and plot of the far field noise from 10 Hz to 10 kHz in decibels.

- 0° angle of attack
- Turbulence intensity, $\overline{u^2}/U_\infty^2 = 11\%$
- chord, c=0.5 m
- span, b=0.1 m
- longitudinal turbulence integral lengthscales = 30 mm, 60 mm, 120 mm
- U_∞=64 m/s

Solution Problem 14.1

The far field sound radiated by a blade is related to the surface pressure difference on that blade through equation 14.2.1.

$$\tilde{p}(\mathbf{x},\omega) \approx -\frac{i\pi\omega x_2 e^{ik_o r_e - ik_o M x_1}}{c_\infty r_e^2}\Delta\tilde{\tilde{p}}(k_1^{(o)},k_3^{(o)},\omega) \quad \text{where } k_1^{(o)} = k_o\left(\frac{x_1}{r_e} - M\right) \text{ and } k_3^{(o)} = \frac{k_o x_3 \beta^2}{r_e}$$

At a frequency ω the sound is produced by a surface pressure difference fluctuation with a spanwise wavenumber $k_3^{(o)} = \frac{k_o x_3 \beta^2}{r_e} = \frac{\omega}{\beta^2 c_\infty}\frac{x_3 \beta^2}{r_e} = \omega\frac{x_3\beta^2}{r_e c_\infty}$ and thus,

$$\left|\frac{\omega}{k_3^{(o)}}\right| = c_\infty\frac{r_e}{|x_3|} = c_\infty\frac{\sqrt{x_1^2+\beta^2(x_2^2+x_3^2)}}{|x_3|} \geq \beta c_\infty$$

Thus the spanwise trace speed of pressure difference disturbances along the leading edge that produce sound is supersonic or nearly supersonic. The surface pressure difference is related to the gust encountered by the blade leading edge via equation 13.4.5,

$$\Delta p^{(1)}(y_1,y_3) = -\frac{2\pi\rho_o U_\infty \hat{a}_2 e^{i(\kappa - Mk_o)y_1 + ik_3y_3 + i\pi/4}}{\pi^{3/2}(k_1+\kappa\beta^2)^{1/2}y_1^{1/2}} \qquad y_1 > 0$$

where $k_1 = \omega/U$, $\kappa = \left(k_o^2 - \frac{k_3^2}{\beta^2}\right)^{1/2}$, $k_o = k_1 M/\beta^2$ and $\beta^2 = 1 - M^2$. Note that this equation shows that the pressure difference on the airfoil is spanwise periodic with the same wavenumber as that of the flow disturbance k_3 and thus k_3 in equation 13.4.5 is equivalent to $k_3^{(o)}$ in equation 14.2.1. Thus the sound radiating pressure fluctuations are those with,

$$\left|\frac{\omega}{k_3}\right| \geq \beta c_\infty$$

or

$$\frac{\omega^2}{k_3^2} \geq \beta^2 c_\infty^2 \quad\rightarrow\quad \frac{\omega^2/c_\infty^2}{(1-M^2)^2} \geq \frac{k_3^2}{1-M^2} \quad\rightarrow\quad \frac{\omega^2/c_\infty^2}{(1-M^2)^2} \geq \frac{k_3^2}{1-M^2} \quad\rightarrow\quad k_o^2 \geq \frac{k_3^2}{\beta^2}$$

This is the condition for κ to be real. In this case the exponent in the surface pressure jump of 13.4.5 is entirely imaginary and a surface pressure disturbance generated at the leading edge will propagate over the blade, attenuating only slowly downstream with $y_1^{-1/2}$. This is the footprint of the soundwave radiating away from the airfoil surface above and below.

When the trace spanwise trace speed of a gust along the leading edge is subsonic, then κ is imaginary and the exponent gains a real term. We will have

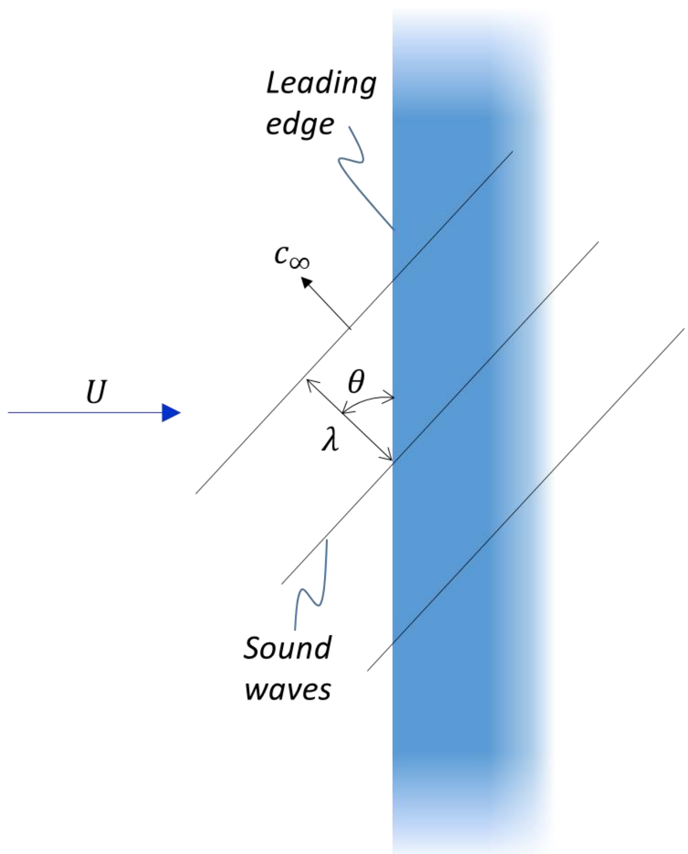
$$\kappa = i\left(\frac{k_3^2}{\beta^2} - k_o^2\right)^{1/2}$$

And thus the κ term of the exponent becomes

$$e^{i\kappa y_1} = e^{-\left(\frac{k_3^2}{\beta^2} - k_o^2\right)^{1/2} y_1}$$

Now we see that the disturbance decays exponentially downstream of the leading edge. This is the footprint of an aerodynamic disturbance that does not propagate to the far field.

The reason it is possible for the footprint of a sound wave radiating to the far field to have a trace speed down to βc_∞, just below the sound speed, is because of the effects of convection on the sound. Consider the situation shown in the figure. A sound wave of wavelength λ produced by the gust interaction is traveling upstream at angle θ to the leading edge. It's propagating through the air at c_∞ but at the same time being convected downstream at speed U by the flow. The apparent speed of the wave, perpendicular to its wavefronts will therefore be $c_\infty - U\sin\theta$ and this determines the frequency seen at a fixed point,



$$\omega = 2\pi(c_\infty - U\sin\theta)/\lambda$$

The spanwise wavenumber associated with the wave is,

$$k_3 = \frac{2\pi}{\lambda}\cos\theta$$

and thus the trace speed of the sound wave along the leading edge is,

$$\frac{\omega}{k_3} = c_\infty \frac{1 - M\sin\theta}{\cos\theta}$$

The multiplier on the sound speed can be less than 1. Indeed, its minimum value is $\sqrt{1 - M^2} = \beta$.