Chapter 3

3.1 A faulty air handling system in Blacksburg (a town 2000 feet above sea level) generates piercing tones (sound at a discrete frequency) at 2200Hz and 3800Hz with amplitudes recorded at a test microphone of 5.2 and 3.7 Pascals, respectively. Determine the (a) the RMS acoustic pressure, (b) the SPL in dB, (c) the acoustic intensity assuming the microphone is in the far field. Give units with your answers.

3.2 Write a short Matlab program to generate and compare contour plots of the pressure field generated by two monopoles of opposite strength placed at $y_1 = -d/2$ and $+d/2$ (Figure 3.4) and the equivalent dipole. Plot the pressure field within $5d$ of the origin and choose 3 values of kd: 2, 4 and 8. Note that these represented conditions: (a) where the monopole pair is acoustically compact and acoustic far-field is reached within the $5d$ range; (b) where the monopole pair is compact but the near field is clearly visible; (c) where the monopole pair is not compact. Comment on the comparison between the dipole and monopole pair sound fields in each of these cases.

3.3 Consider noise made by a submarine 100m in length in an otherwise unbounded ocean as heard by a fixed observer in the far field who is looking at the bow along the axis of the sub. Suppose there are known sound sources at the bow and stern. (a) With the submarine stationary, what will be the difference in the retarded time for these two sources? What phase difference will that correspond to at a frequency of 1kHz? (b) Repeat part (a) for the case where the submarine is traveling at a steady 40 knots towards an observer. (c) For the conditions given in part (b), over what frequency range would we need to account for effect of the vehicle motion on the retarded time to accurately predict the sound heard by the observer, assuming we can tolerate an error in phase of 10 degrees?

3.4 Find and simplify an expression for the far field acoustic intensity produced by a complex non-compact source consisting of a series of N monopoles of different amplitude, phase and position in a quiescent medium. Your answer should be in terms of the ambient density, sound speed, observer positions and the complex monopole amplitudes. How does this expression change if the source is acoustically compact? Interpret this result.

3.5 For a fixed observer location **x** and fixed time *t* show that the free field Greens function given by equation 3.9.17 represents a collapsing impulsive wave onto the point **x** arriving at time *t* when $t<\tau$ and is zero when $t>\tau$.
Worked example solution

3.6. For a fixed location **y** and time τ show that the free field Greens function given by equation 3.9.17 represents an outgoing impulsive wave from the point **y** initiated at time τ when $t>\tau$ and is zero when $t<\tau$, and hence that equation 3.9.12 represents waves propagating away from the surface *S*.
Worked example solution

3.7 Obtain the solution to equation 3.9.2 in the free field by considering the trial solution

$$G_o(\mathbf{x},t|\mathbf{y},\tau) = \frac{f(\tau \pm r/c_o)}{r}$$

and making use of the identities

$$\nabla^2\big(f(r)\big) = \frac{\partial^2 f(r)}{\partial r^2} + \frac{2}{r}\frac{\partial f(r)}{\partial r} \qquad \nabla^2(1/r) = -4\pi\delta(\mathbf{x}-\mathbf{y}) \qquad r = |\mathbf{x}-\mathbf{y}|$$

Worked example solution

3.8 The exterior boundary in the surface integral in equation 3.9.12 was not included because of the Sommerfeld radiation condition. Show that this is valid.
Worked example solution

3.9 An underwater vehicle generates tonal noise recorded by a far field observer. The vehicle generates two tones, one with an SPL (Re 20μPa) of 68dB at 150 Hz, and the second with an SPL of 64dB at 200Hz. You make take the speed of sound in the ocean to be 1500 m/s and the water density to be 1027 kg/cubic meter.

(a) What is the SPL of the overall sound heard by the observer?

(b) How small must the noise generating device be to form a compact source at 150Hz?

(c) It is determined that the source of sound from at 150Hz is a compact monopole. Estimate the volume displacement amplitude Q of this source given that the observer is 100m from the vehicle. State the units of your answer.

Worked example solution

Solution Problem 3.5

When $t \neq \tau$ equation 3.9.2 is the homogeneous wave equation and has the solution, for fixed $\mathbf{x}$ and t, given by

$$G_o(\mathbf{x}, t|\mathbf{y}, \tau) = \frac{f(\tau - r/c_o)}{r} + \frac{g(\tau + r/c_o)}{r} \qquad r = |\boldsymbol{x} - \boldsymbol{y}| > 0 \qquad t \neq \tau$$

where $f(\tau - r/c_o)/r$ is a wave propagating outwards from the point $\mathbf{x}$ and $g(\tau + r/c_o)/r$ is a wave collapsing onto the point $\mathbf{x}$. Equation 3.9.17 gives

$$G_o(\mathbf{x}, t|\mathbf{y}, \tau) = \frac{\delta(t - (\tau + r/c_o))}{4\pi r} \qquad r = |\boldsymbol{x} - \boldsymbol{y}|$$

and so, for a fixed t, represents a collapsing wave. It follows that G_o is only non zero when $t = \tau + r/c_o$ and since $r > 0$ by definition, this is only possible if $\tau < t$.

Solution Problem 3.6

Equation 3.9.17 gives

$$G_o(\mathbf{x}, t|\mathbf{y}, \tau) = \frac{\delta((t - r/c_o) - \tau)}{4\pi r} \qquad r = |\boldsymbol{x} - \boldsymbol{y}|$$

and for a fixed τ represents an outgoing wave from the point $\mathbf{y}$. It also follows that G_o is only non zero when $t - r/c_o = \tau$ and since $r > 0$ by definition this is possible only if $\tau < t$. All points on the surface S are defined as a function of $\mathbf{y}$ and so the left side of 3.9.12 represents waves propagating away from the surface.

Solution Problem 3.7

Given that equation 3.9.2 is

$$\frac{1}{c_o^2}\frac{\partial^2 G_o}{\partial \tau^2} - \nabla^2 G_o = \delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)$$

(with $\mathbf{x}$ and t fixed, and the Laplacian defined as a function of $\mathbf{y}$) we substitute the trail solution to obtain

Eq. (A)

$$\frac{1}{rc_o^2}\frac{\partial^2 f(\tau \pm r/c_o)}{\partial \tau^2} - \nabla^2\left(\frac{f(\tau \pm r/c_o)}{r}\right) = \delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)$$

then since

$$\nabla^2\left(\frac{f(\tau \pm r/c_o)}{r}\right) = \frac{1}{r}\nabla^2\big(f(\tau \pm r/c_o)\big) + 2\nabla\big(f(\tau \pm r/c_o)\big).\nabla\left(\frac{1}{r}\right) + \nabla^2\left(\frac{1}{r}\right)f(\tau \pm r/c_o)$$

$$= \frac{1}{r}\left(\frac{\partial^2}{\partial r^2}\big(f(\tau \pm r/c_o)\big) + \frac{2}{r}\frac{\partial}{\partial r}\big(f(\tau \pm r/c_o)\big)\right) - \frac{2}{r^2}\frac{\partial}{\partial r}\big(f(\tau \pm r/c_o)\big) - 4\pi\delta(\mathbf{x}-\mathbf{y})f(\tau \pm r/c_o)$$

$$= \frac{1}{r}\left(\frac{\partial^2}{\partial r^2}\big(f(\tau \pm r/c_o)\big)\right) - 4\pi\delta(\mathbf{x}-\mathbf{y})f(\tau \pm r/c_o)$$

$$= \frac{1}{rc_o^2}\left(\frac{\partial^2}{\partial \tau^2}\big(f(\tau \pm r/c_o)\big)\right) - 4\pi\delta(\mathbf{x}-\mathbf{y})f(\tau \pm r/c_o)$$

Substituting this into Eq. (A) gives

$$4\pi\delta(\mathbf{x}-\mathbf{y})f(\tau \pm r/c_o) = \delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)$$

This equation has a trivial solution unless $\mathbf{x}=\mathbf{y}$ or *r=0* and so

$$4\pi\delta(\mathbf{x}-\mathbf{y})f(\tau) = \delta(\mathbf{x}-\mathbf{y})\delta(t-\tau)$$

Hence it follows that $f(\tau \pm r/c_o) = \delta(t-\tau \pm r/c_o)/4\pi$ and hence that $G_o = \delta(t-\tau \pm r/c_o)/4\pi r$. If we also impose the condition that $G_o = 0$ when $\tau > t$ so that the wave motion is causal then only the negative sign in the argument satisfies the condition and we obtain equation 3.9.17.

Solution Problem 3.8

Consider an exterior surface S_e that encloses the point $\mathbf{x}$ and all the interior surfaces shown in Figure 3.7. Define a second surface S_∞ that encloses S_e and is a sphere centered on $\mathbf{x}$ with radius $r_\infty > (t+T)c_o$. Find the solution to the wave equation in the volume V_e that is defined as being between the surfaces S_e and S_∞ *for the point* $\mathbf{x}$ *that lies inside* S_e and S_∞. When integrating equation 3.9.7 the point $\mathbf{x}$ is not in V_e and so the right side of 3.9.8 is zero, so

$$\int_{V_e} \int_{-T}^{T} \frac{1}{c_o^2}\left(p'\frac{\partial^2 G_o}{\partial \tau^2} - G_o\frac{\partial^2 p'}{\partial \tau^2}\right) - \left(p'\frac{\partial^2 G_o}{\partial y_i^2} - G_o\frac{\partial^2 p'}{\partial y_i^2}\right) d\tau dV(\mathbf{y}) = 0$$

Then, as in equation 3.9.10, providing p' and it's time derivative are zero for $\tau<-T$ and the causality condition applies, the first term in the integrand integrates to zero and we can evaluate the volume integral so that, as in 3.9.12,

$$\int_{-T}^{T} \int_{S_e+S_\infty} \left(p'(\mathbf{y},\tau)\frac{\partial G_o(\mathbf{x},t|\mathbf{y},\tau)}{\partial y_i} - G_o(\mathbf{x},t|\mathbf{y},\tau)\frac{\partial p'(\mathbf{y},\tau)}{\partial y_i}\right) n_i dS(\mathbf{y}) d\tau = 0$$

The two surface integrals either exactly cancel or are zero. However on S_∞ we have that $|\mathbf{x}-\mathbf{y}|=r_\infty>(t+T)c_o$ and so the argument of the Greens function is

$t-\tau-r_\infty/c_c<-(\tau+T)$

The lower limit of the integral is $\tau=-T$ and so the integrand on S_∞ is zero because there are no values of τ that are solutions to $t-\tau-r_\infty/c_c=0$ and hence the Greens function $G_o=\delta(t-\tau-r_\infty/c_c)/4\pi r_\infty=0$ on S_∞. It follows that the integral over S_e is also zero, proving the assertion that the exterior boundary makes no contribution to the result given by equation 3.9.12.

Solution Problem 3.9

(a) $SPL = 10\log_{10}\left(\frac{\overline{p^2}}{(20\mu Pa)^2}\right)$

For $150Hz$ tone $\overline{p^2} = 10^{6.8} \times 4 \times 10^{-10} = 0.002524\ Pa^2$

For $200Hz$ tone $\overline{p^2} = 10^{6.4} \times 4 \times 10^{-10} = 0.001005\ Pa^2$

So $SPL = 10\log_{10}\left(\frac{0.003529}{(20\mu Pa)^2}\right) = 69.5\ dB$

(b) We require that the object dimension $d \ll \frac{1}{k} = \frac{c_o}{2\pi f} = \frac{1500}{2\pi 150} = 1.59$. Thus $d \ll 1.59\ m$.

(c) From equation 3.4.6, $\hat{p} = -\frac{i\omega\rho_o Q e^{ikr}}{4\pi r}$. Now, $|\hat{p}| = \sqrt{2 \times 0.002524} = 0.071\ Pa$.

So, $|Q| = \frac{4\pi r|\hat{p}|}{\omega\rho_o} = \frac{2\times100\times0.071}{150\times1027} = 9.22 \times 10^{-5}\ m^3/s$