Chapter 1

1.1 Consider a narrow band acoustic spectrum expressed in terms of angular frequency $G_{pp}(\omega)$ where the mean square fluctuation is divided up by its contribution to 1 rad/s intervals (i.e. the spectral density per rad/s). We can define the sound pressure level in decibels for this spectral density as $10\log_{10}\left(\frac{G_{pp}(\omega)}{p_{ref}^2/(rad/s)}\right)$ where, for air $p_{ref}^2 = 4 \times 10^{-10}\ (Pa)^2$. If $G_{pp}(\omega)$ varies with frequency as B/ω^2, and $B = 25\ (Pa)^2/s$, what is the overall A-weighted sound pressure level? *Hint: you will need to research the formula for A-weighting.* Worked example solution

1.2 A noise source has a narrowband sound spectrum (1 Hz intervals) which varies as $-20\log_{10}(f) + 140$ where f is in Hz. This equation represents the spectral density per Hz in decibels ($dB(Hz) = 10\log_{10}((p_{rms}^2/Hz)/(p_{ref}^2/Hz))$) with a reference pressure of 20 μPa. This equation is defined only in the range 100 Hz to 10 kHz. Assume that sound is not produced outside of this range. Compute and plot the 1/3rd octave-band spectra for this narrowband spectrum and the overall sound pressure level in decibels.

1.3. A jet engine produces tones (single frequency sound waves) at integer multiples of 200Hz each with an SPL of 10dB, relative to $20\mu Pa$. (a) Plot the 1/3rd octave SPL spectrum from 200Hz to 20kHz, (b) plot the A-weighted 1/3rd octave spectrum, (c) calculate the overall SPL and the overall A-weighted SPL. Your plots should use a logarithmic frequency scale. Explain why 1/3rd octave band spectra are unlikely to be useful in general for representing tone-noise sources. *Hint: you will need to research the formula for A-weighting.*

Solution Problem 1.1

$$G_{pp}(\omega) = B/\omega^2$$

A-weighting (see https://en.wikipedia.org/wiki/A-weighting) is

$$A(f) = 20\log_{10} R_A(f) + 2.00$$

or

$$A(f) = 10\log_{10}(1.259^2 R_A(f)^2)$$

where

$$R_A(f) = \frac{12194^2 f^4}{(f^2 + 20.6^2)(f^2 + 12194^2)\sqrt{(f^2 + 107.7^2)(f^2 + 737.9^2)}}$$

Now, we have per (rad/s) that

$$SPL = 10\log_{10}\left(\frac{G_{pp}(\omega)}{4 \times 10^{-10}}\right)$$

and

$$SPL_A = 10\log_{10}\left(\frac{G_{pp}(\omega)}{4 \times 10^{-10}}\right) + 10\log_{10}(1.259^2 R_A(f)^2)$$

$$= 10\log_{10}\left(\frac{1.259^2 R_A(f)^2 G_{pp}(\omega)}{4 \times 10^{-10}}\right)$$

where $f = 2\pi\omega$. Thus,

$$OASPL_A = 10\log_{10}\left[\int_0^\infty \frac{1.259^2 R_A(f)^2 G_{pp}(\omega)}{4 \times 10^{-10}} d\omega\right]$$

$$= 10\log_{10}\left[\frac{1.259^2}{4 \times 10^{-10}}\int_0^\infty \frac{12194^4 f^8 \times 25/(2\pi f)^2}{(f^2 + 20.6^2)^2(f^2 + 12194^2)^2(f^2 + 107.7^2)(f^2 + 737.9^2)} df\right]$$

Since, $G_{pp}(\omega)d\omega = G_{pp}(f)df$, so

$$OASPL_A = 10\log_{10}\left[\frac{12194^4 \times 1.259^2 \times 25}{16\pi^2 \times 10^{-10}}\int_0^\infty \frac{f^6}{(f^2 + 20.6^2)^2(f^2 + 12194^2)^2(f^2 + 107.7^2)(f^2 + 737.9^2)} df\right]$$

Using Wolfram Alpha, the integral is evaluated as 7.52482×10^{-20}, so

$$OASPL_A = 10\log_{10}\left[\frac{12194^4 \times 1.259^2 \times 25}{32\pi^3 \times 10^{-10}} \times 7.52482 \times 10^{-20}\right] = 66.2\ dB(A)$$

Note that A-weighting is only defined for 20Hz to 20 kHz, but contributions outside this range to the above integral are negligible. Also note that a conventional approach (producing almost the same answer) would be to first calculate the 1/3rd octave band SPL from Gpp, then A-weight these values at the band center frequencies, and then integrate the mean-square pressures implied by these weighted values to get the final OASPLA.