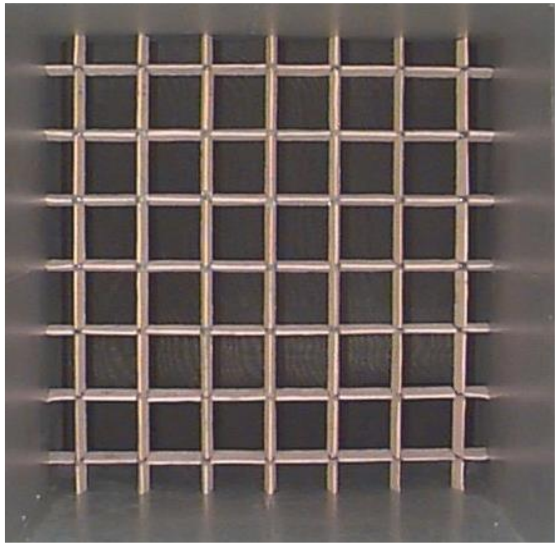
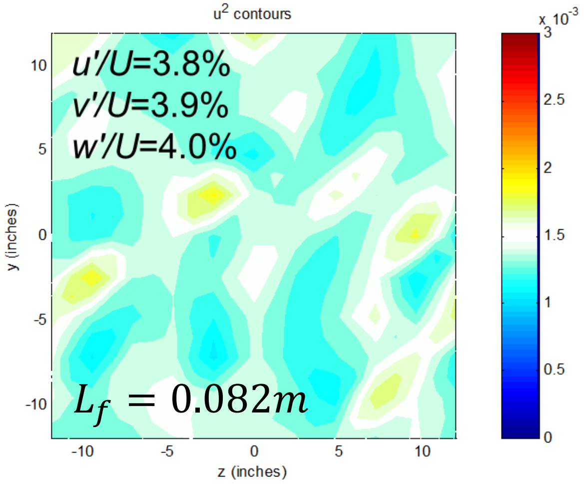
Example: The 2010 VT Airfoil Experiment

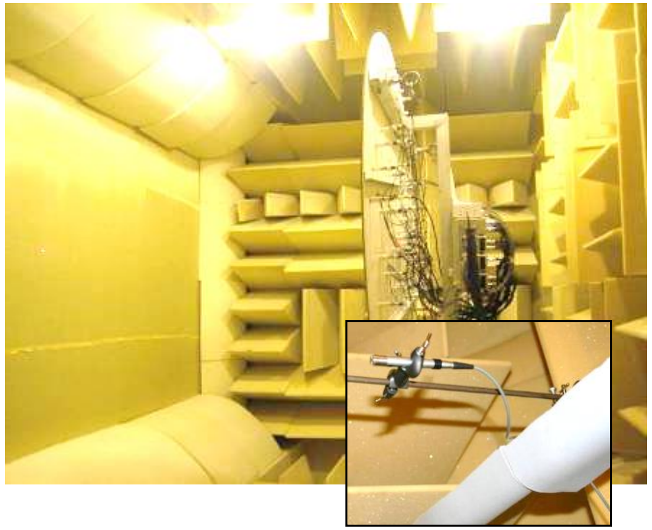
Devenport, W. J., J. K. Staubs and S. A. L. Glegg (2010). "Sound radiation from real airfoils in turbulence." Journal of Sound and Vibration **329(17): 3470-3483.**



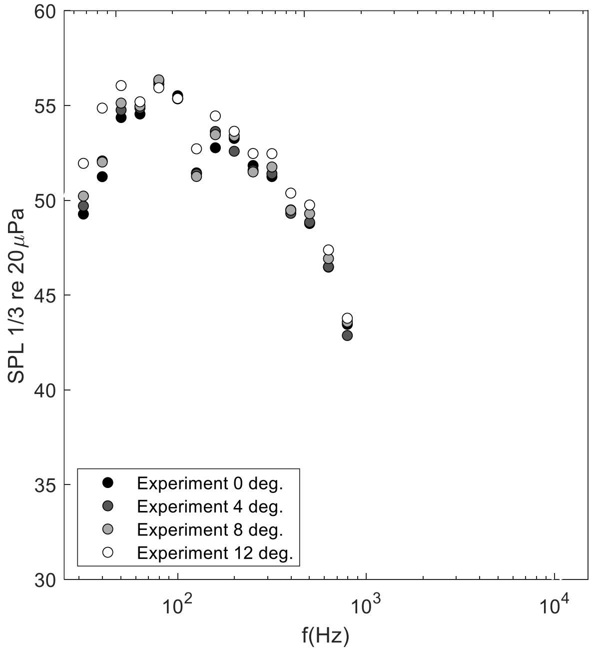


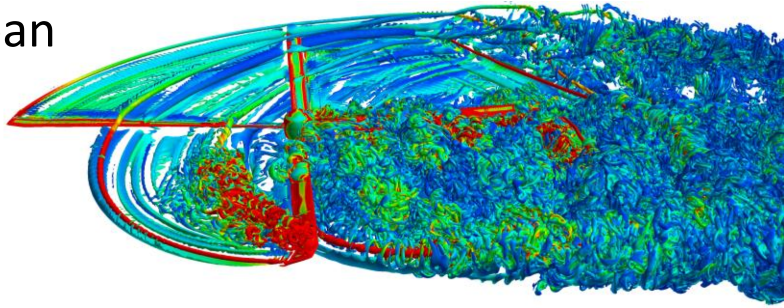
NACA 0012
0.2023-m chord
1.854-m span

$U_\infty = 30\ m/s$



B&K ½-inch mic at 1.8m directly 'above' center span (results scaled to 10-m observer distance





Example

1.854-m span 0.2032-m chord blade immersed in a steady turbulent flow with $U_\infty = 30m/s$, $\sqrt{\overline{u_2^2}}/U_\infty = 3.9\%$, $L_f = 0.082m$. Plot the sound spectrum in SPL$_{1/3}$ heard by an observer in the frame of reference of the blade 10m 'above' the blade midspan leading edge.

$$S_{pp}(x,\omega) = \frac{\pi\rho_o^2 U_\infty b c^2 k^2}{8 r_g^2}\frac{x_2^2}{r_g^2}\phi_{22}\left(\frac{\omega}{U_\infty}, k_3^{(o)}\right)\left|\Lambda\left(k_1^{(o)}, k_3^{(o)}, \omega, M\right)\right|^2$$

Assuming von Karman spectrum, but could assume Liepmann (see eqn 11.3.23), or use measured wake data, see aeroacoustics.net

$$\Lambda(k_1, k_3, \omega, M) \approx \frac{-2(1+i)E_2\big((\kappa - Mk_o - k_1)c\big)}{\pi(\omega/U_\infty + \kappa\beta^2)^{1/2}\,(\kappa - Mk_o - k_1)^{1/2}c}$$

Using compressible flat plate response function (ignoring thickness)

$$\phi_{22}(k_1,k_3) = \frac{4}{9\pi}\frac{\overline{u^2}}{k_e^2}\frac{(k_1^2+k_3^2)/k_e^2}{[1+(k_1^2+k_3^2)/k_e^2]^{7/3}}$$

Plotting results scaled to 1/3rd octave SPL:

$$SPL_{1/3} = 10\log_{10}\left(\frac{2S_{pp}\omega}{(20\mu Pa)^2}[2^{1/6} - 2^{-1/6}]\right)$$

$$r_g^2 = x_1^2 + \beta^2 x_2^2 + \beta^2 x_3^2$$

$$k_1^{(o)} \equiv \frac{\omega}{\beta^2 c_\infty}\left(\frac{x_1}{r_g} - M\right) \quad k_3^{(o)} \equiv \frac{\omega}{c_\infty}\left(\frac{x_3}{r_g}\right)$$

Simple to evaluate Fresnel integral E_2 in Matlab:

```
function res=E2(x)
res=fresnelc(sqrt(2*x/pi))+i*fresnels(sqrt(2*x/pi));
end
```

$$\kappa \equiv (k_o^2 - k_3^2/\beta^2)^{1/2} \qquad k_o \equiv \omega/\beta^2 c_\infty$$

$$k_e = \frac{\sqrt{\pi}}{L_f}\frac{\Gamma\left(5/6\right)}{\Gamma\left(1/3\right)}$$

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```
clear all;
rho=1.1;cinf=340;uinf=30;u2=(uinf*0.039)^2;lf=0.082;
c=0.2032;b=1.8542;m=uinf/cinf;beta=sqrt(1-m^2);
f=logspace(1,4,100);omega=f*2*pi;k=omega/cinf;

x1=0;x2=10;x3=0;
rg=sqrt(x1^2+beta^2*(x2^2+x3^2));
k10=k/beta^2*(x1/rg-m);k30=k*x3/rg;
k0=k/beta^2;kappa=sqrt(k0.^2-k30.^2/beta^2);
ke=sqrt(pi)/lf*gamma(5/6)/gamma(1/3);

phi22=4/9/pi*u2^2/ke^2*(omega.^2/uinf^2+k30.^2)/ke^2./(1+(omega.^2/uinf^2+
k30.^2)/ke^2).^(7/3);
lambda=-2*(1+i)*E2((kappa-m*k0-
k10)*c)/pi./sqrt(omega/uinf+kappa*beta^2)./sqrt(kappa-m*k0-k10)/c;
spp=pi*rho^2*uinf*b*c^2*k.^2/8/rg^2*x2^2/rg^2.*phi22.*abs(lambda).^2;
spl3=10*log10(2*omega.*(2^(1/6)-2^(-1/6)).*spp/4e-10);

figure;h=semilogx(f,spl3,'k');
xlabel('f(Hz)');ylabel('SPL 1/3 re 20\muPa');
```

$$S_{pp}(x,\omega) = \frac{\pi\rho_o^2 U_\infty b c^2 k^2}{8 r_g^2}\frac{x_2^2}{r_g^2}\phi_{22}\left(\frac{\omega}{U_\infty}, k_3^{(o)}\right)\left|\Lambda\left(k_1^{(o)}, k_3^{(o)}, \omega, M\right)\right|^2$$

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$$k_1^{(o)} \equiv \frac{\omega}{\beta^2 c_\infty}\left(\frac{x_1}{r_g} - M\right) \qquad k_3^{(o)} \equiv \frac{\omega}{c_\infty}\left(\frac{x_3}{r_g}\right)$$

$$\kappa \equiv (k_o^2 - k_3^2/\beta^2)^{1/2} \qquad k_o \equiv \omega/\beta^2 c_\infty$$

$\rho_o = 1.1\ kg/m^3$, $c_\infty = 340$

$$k_e = \frac{\sqrt{\pi}}{L_f}\frac{\Gamma\left(5/6\right)}{\Gamma\left(1/3\right)}$$

Example

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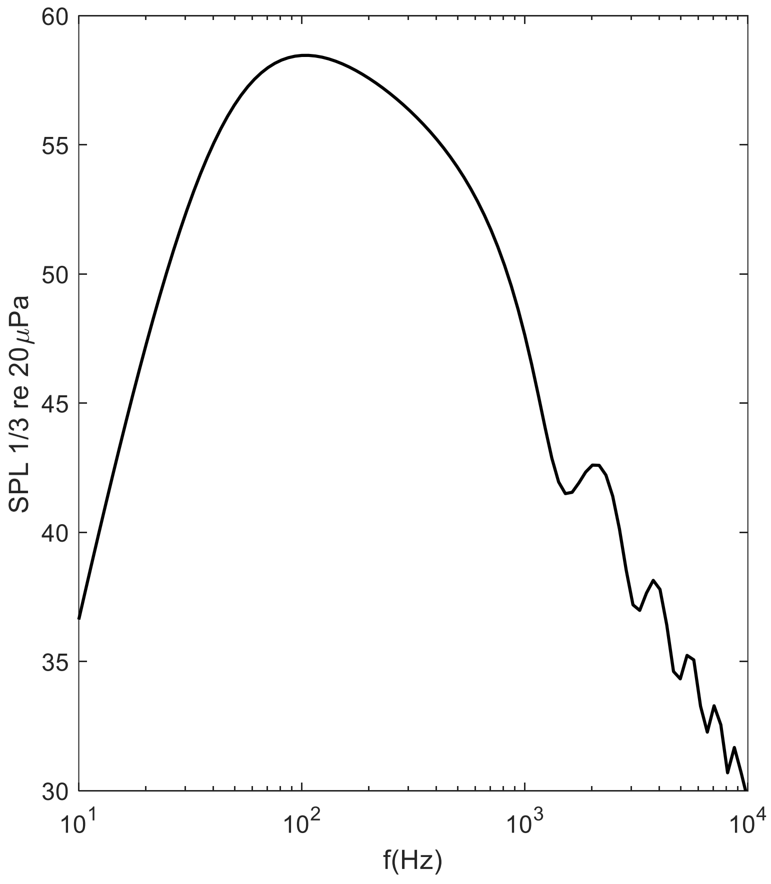

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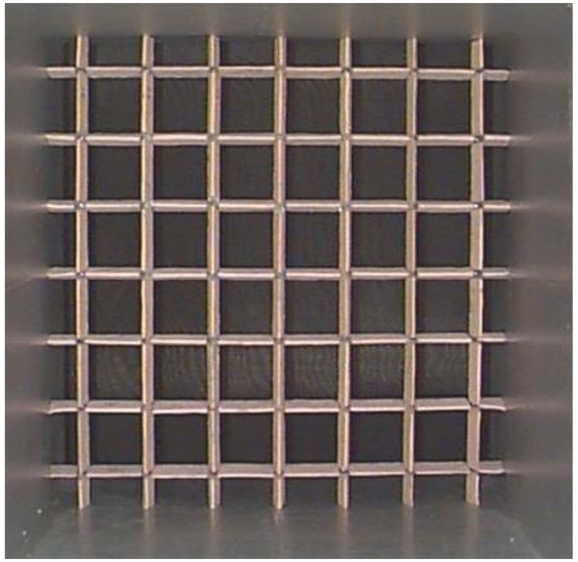

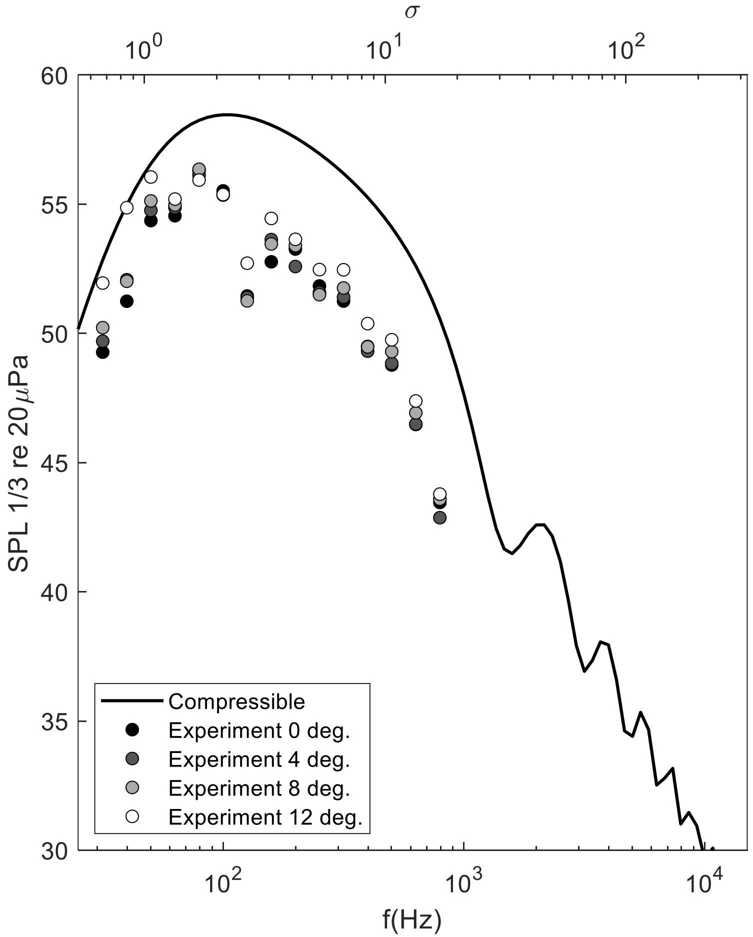

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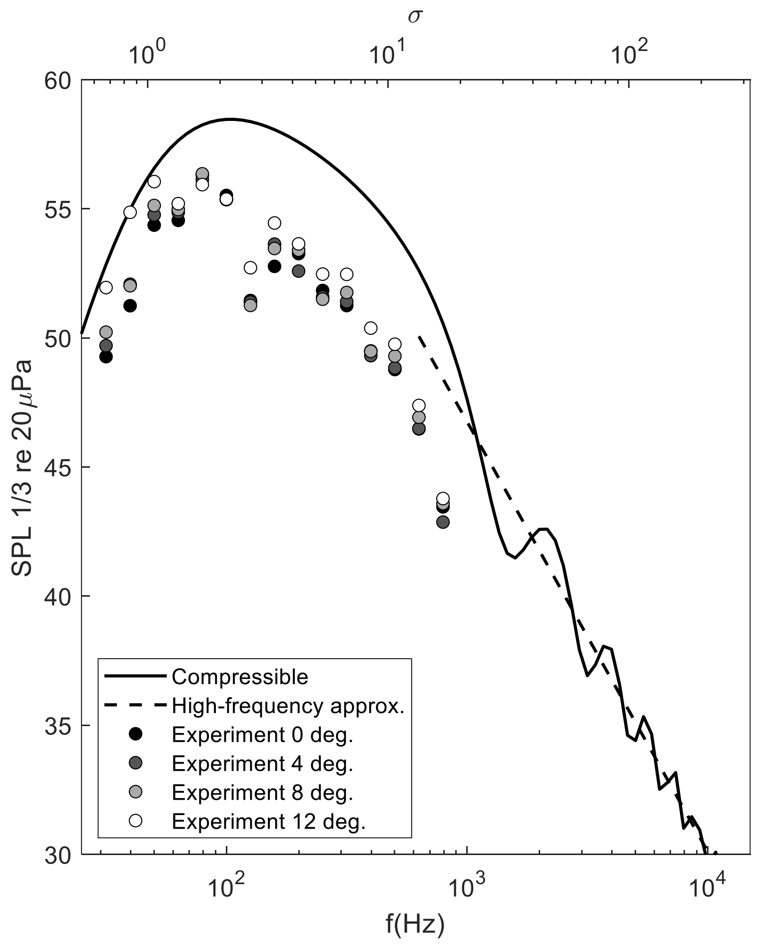
Also…

- High-frequency approximation 13.2.5



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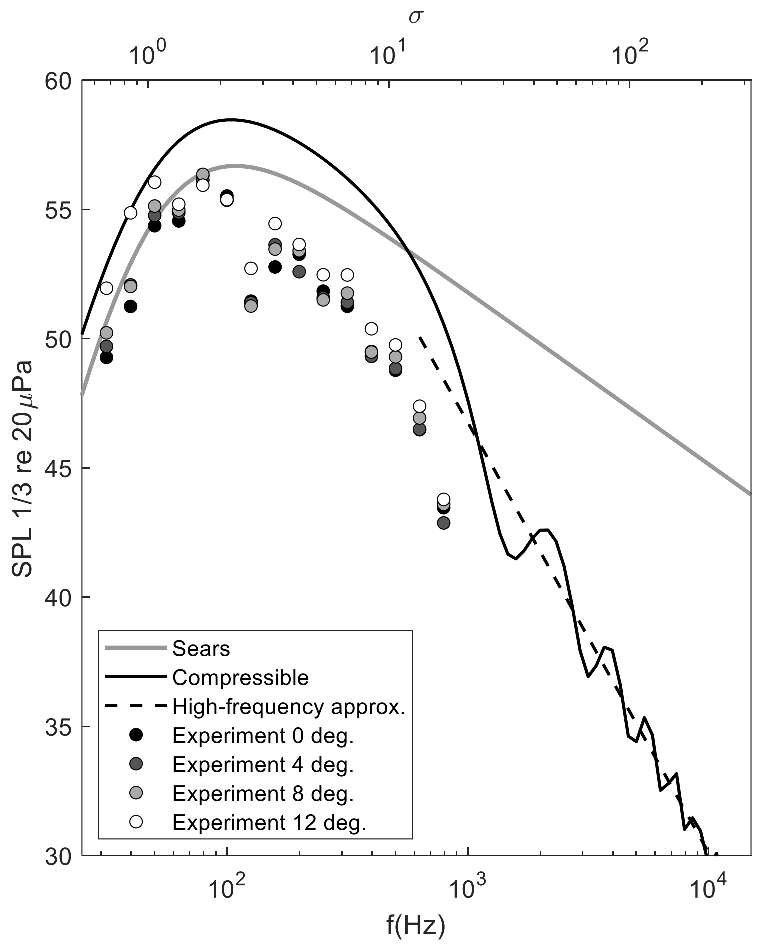
- High-frequency approximation 13.2.5
- Sears (incompressible response), 13.2.7



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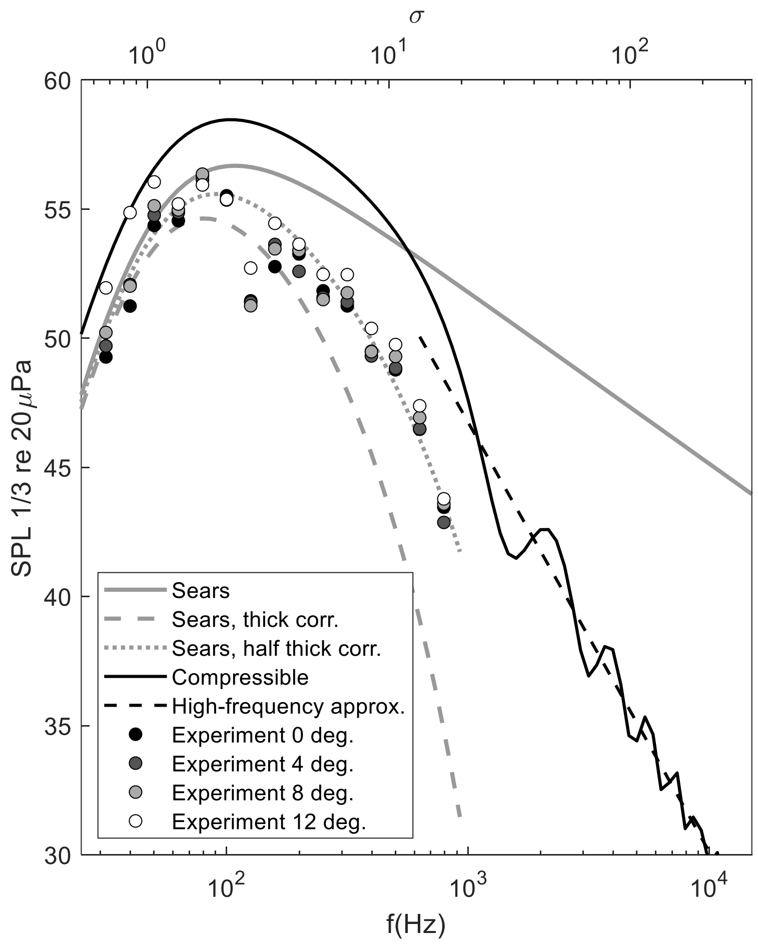
Also…

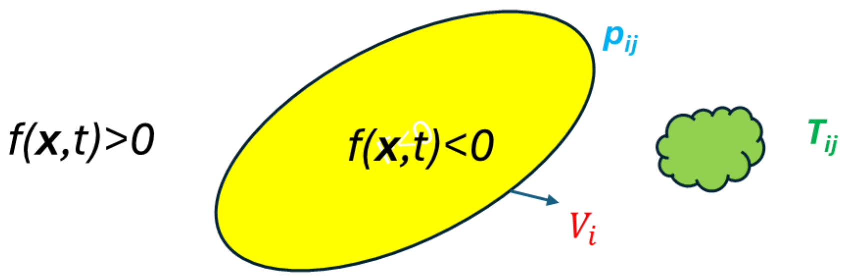
- High-frequency approximation 13.2.5
- Sears (incompressible response), 13.2.7
- Thickness correction, section 9.6.2



Some closing thoughts

Key Results - Lighthill's Theory

- Lighthill's wave equation is an Analogy
- Sound from free turbulence scales as U^8 (hot jets scale as U^6)
- Noise from surfaces in a flow is caused by Thickness noise, Loading noise and Quadrupole noise
- Loading noise scales as U^6 (or U^5 for edge noise sources)
- Sources moving supersonically can reverse the source signature at the observer.



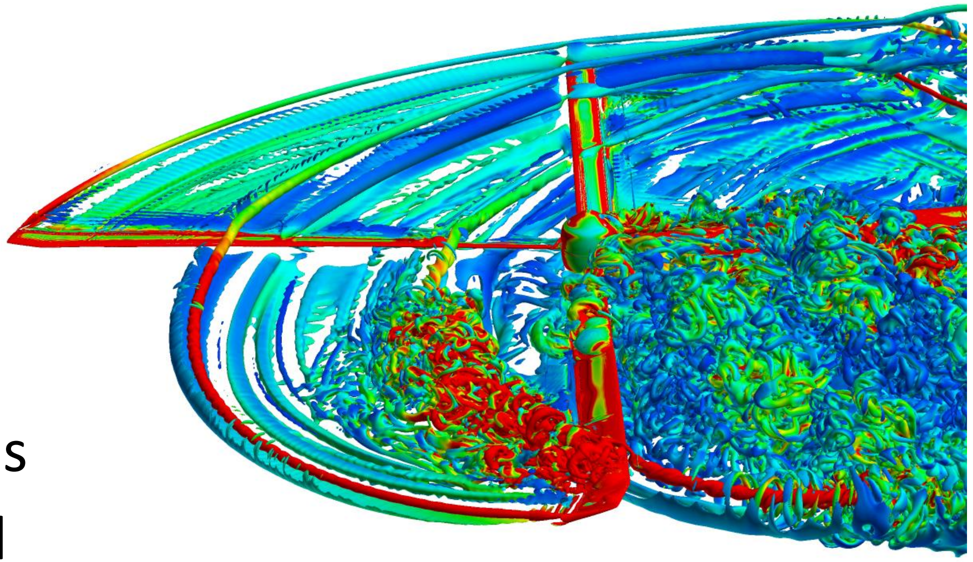
$$\rho' c_\infty^2 H_s = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \left[\frac{T_{ij} H_s}{4\pi r(\tau)|1 - M_r|} \right]_{\tau=\tau^*} dV$$

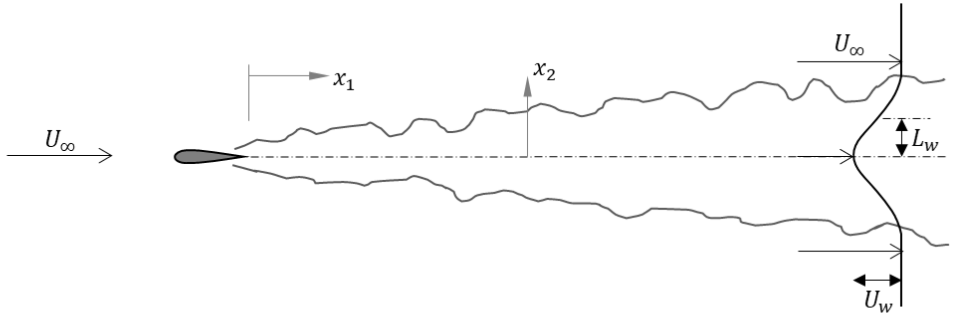
$$- \frac{\partial}{\partial x_i} \int_S \left[\frac{p_{ij} n_j}{4\pi r(\tau)|1 - M_r|} \right]_{\tau=\tau^*} dS$$

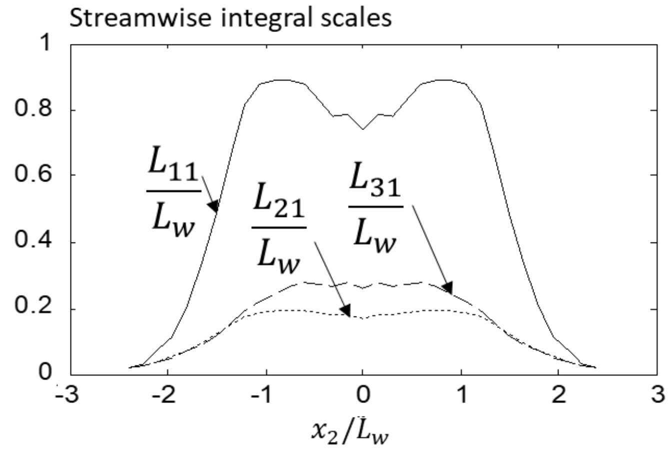
$$+ \frac{\partial}{\partial t} \int_S \left[\frac{\rho_\infty V_i . n_i}{4\pi r(\tau)|1 - M_r|} \right]_{\tau=\tau^*} dS$$

Key Results - Turbulence

- We need 2-point statistics (spatial correlations, wavenumber spectra) to define aeroacoustic sources
- Simple analytical models for spatial correlations and wavenumber spectra exist for homogeneous isotropic turbulence
- These also fit quite well to experimental data from inhomogeneous turbulent flows
- Experimental data is available for the two-point statistics of turbulent wakes and boundary layers
- Analytic expressions exist for the boundary wall pressure frequency spectra, and wavenumber-frequency spectra (needed for TE noise)







Key Results - Open Rotor Noise

- Increasing the rotor diameter for a given thrust reduces the noise
- Doppler factors of $(1 - M_r)$ have a significant impact on directionality
- Rotors have tone noise, BVI and broadband noise
- Inflow distortions increase noise
- Blade response functions determine the loading noise
- A compressible blade response function must be used when the chord is greater than 1/4 of the acoustic wavelength