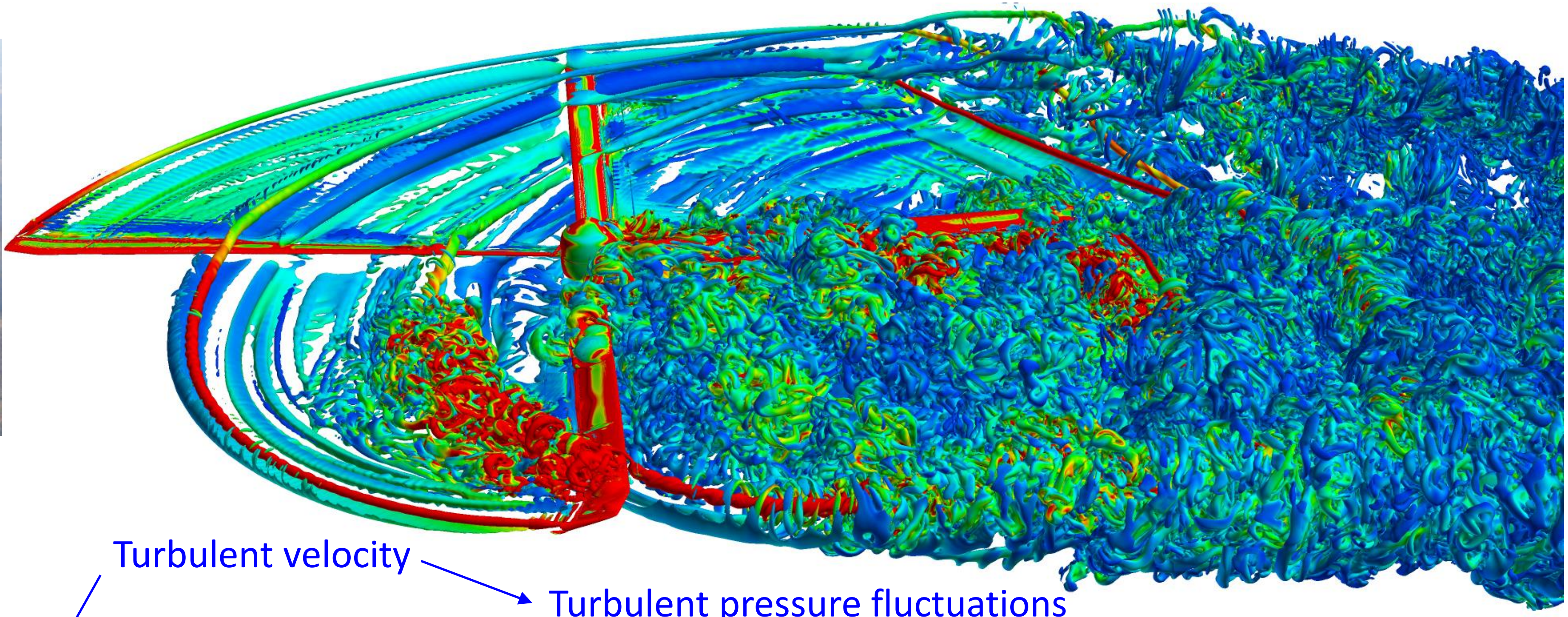
Stochastic Processes and Turbulent Flow



Acoustic pressure

Turbulent velocity

Turbulent pressure fluctuations

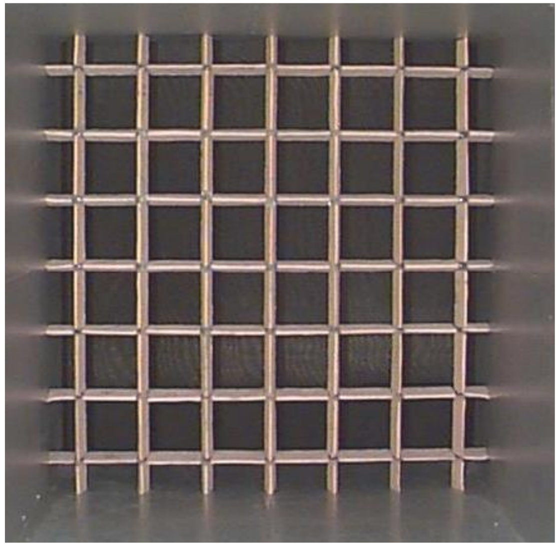
$$\rho' c_\infty^2 H_s = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \left[\frac{T_{ij} H_s}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dV - \frac{\partial}{\partial x_i} \int_S \left[\frac{p_{ij} n_j}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dS + \frac{\partial}{\partial t} \int_S \left[\frac{\rho_\infty V_i . n_i}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dS$$

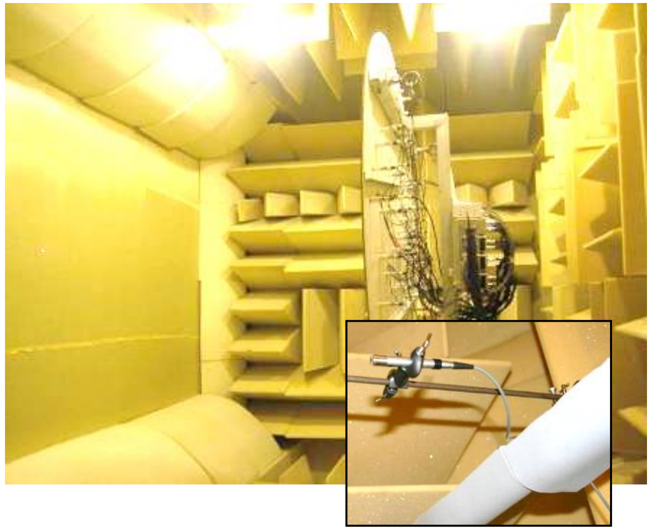
https://en.wikipedia.org/wiki/Sikorsky_UH-60_Black_Hawk

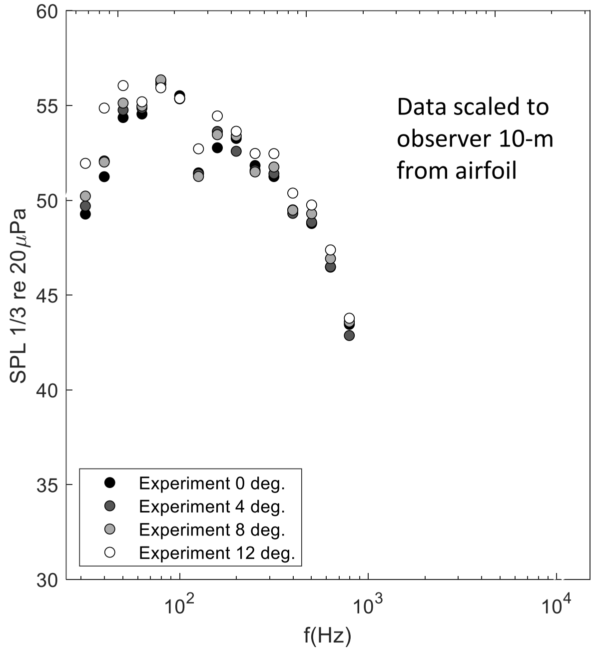
Chaderjian, N. M., (2018). Numerical Simulation of Dynamic Stall Using Near-Body Adaptive Mesh Refinement. 10th International Conference on CFD, Barcelona.

Stochastic Processes and Turbulent Flow

Devenport, W. J., J. K. Staubs and S. A. L. Glegg (2010). "Sound radiation from real airfoils in turbulence." Journal of Sound and Vibration **329(17): 3470-3483.**







Acoustic pressure

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$$\rho' c_\infty^2 H_s = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \left[\frac{T_{ij} H_s}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dV - \frac{\partial}{\partial x_i} \int_S \left[\frac{p_{ij} n_j}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dS + \frac{\partial}{\partial t} \int_S \left[\frac{\rho_\infty V_i . n_i}{4\pi r(\tau)|1 - M_r|}\right]_{\tau=\tau^*} dS$$

- To characterize the sound, we need to know how to characterize the typical behavior of a stochastic process.
- To predict the sound, we need some way to estimate the statistical properties of the turbulence that generates it

Stochastic Processes and Turbulent Flow

Ch 10

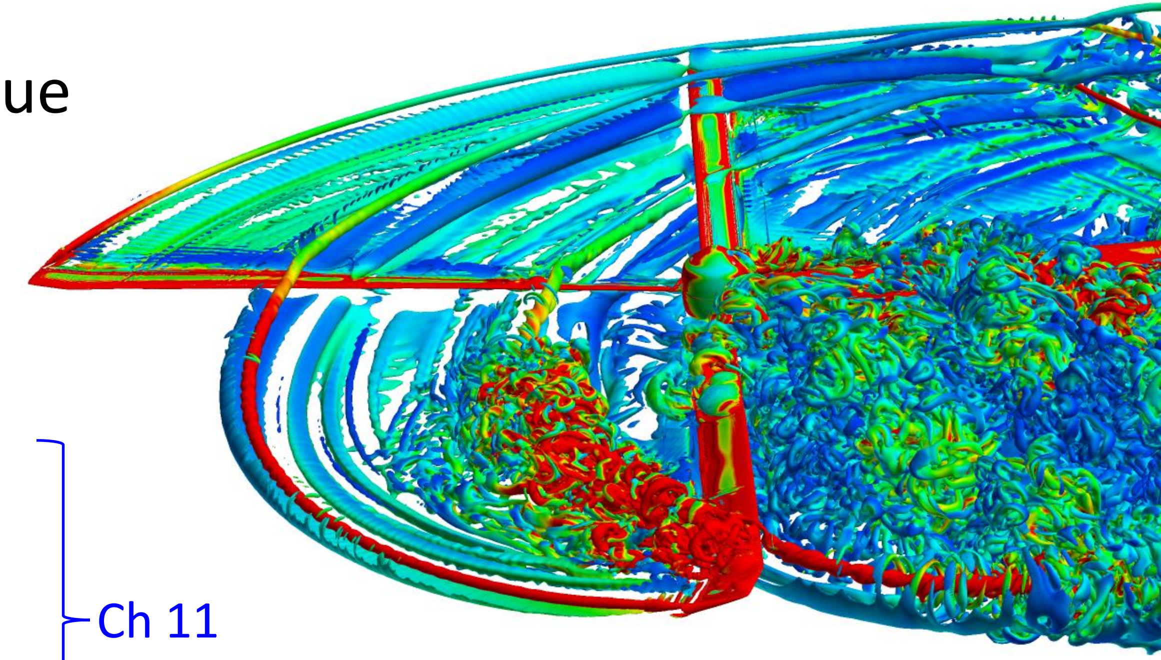
1. Stochastic processes and the expected value
2. Time spectra and correlations
3. Cross correlations and cross spectra
4. Wavenumber spectra
5. Turbulence and aeroacoustics
6. Homogeneous isotropic turbulence
7. The plane wake
8. Zero pressure gradient turbulent boundary layer
9. Leading edge noise example

Ch 11

Ch 12

Ch 13

At the end of the open rotor noise lecture



1. Stochastic processes and the expected value

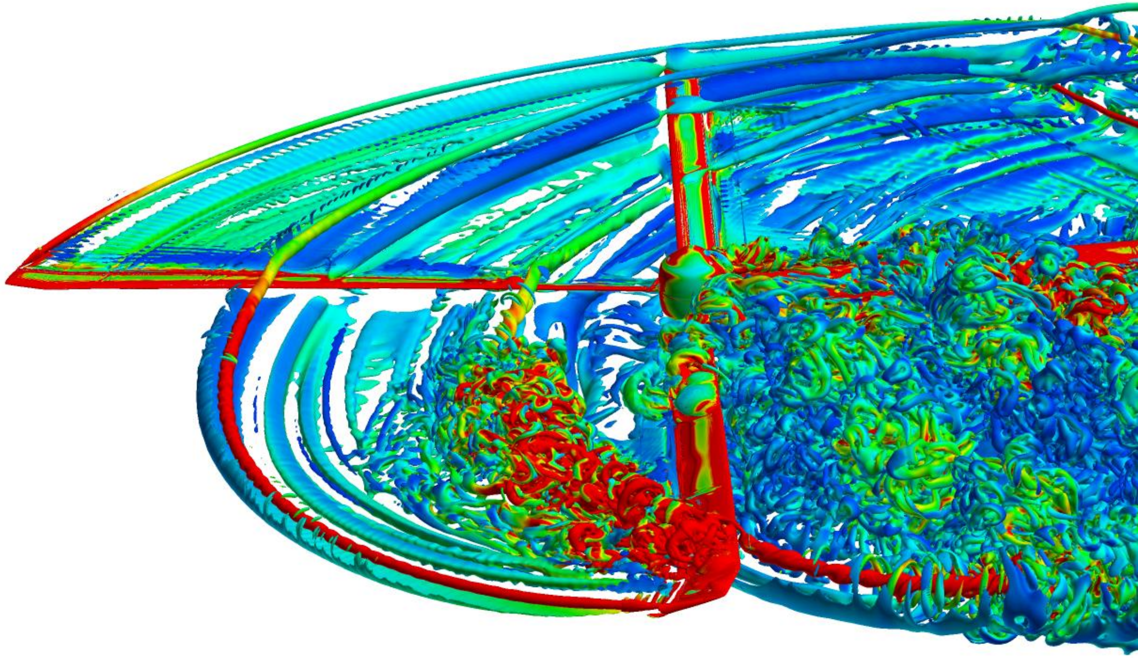
Section 10.1

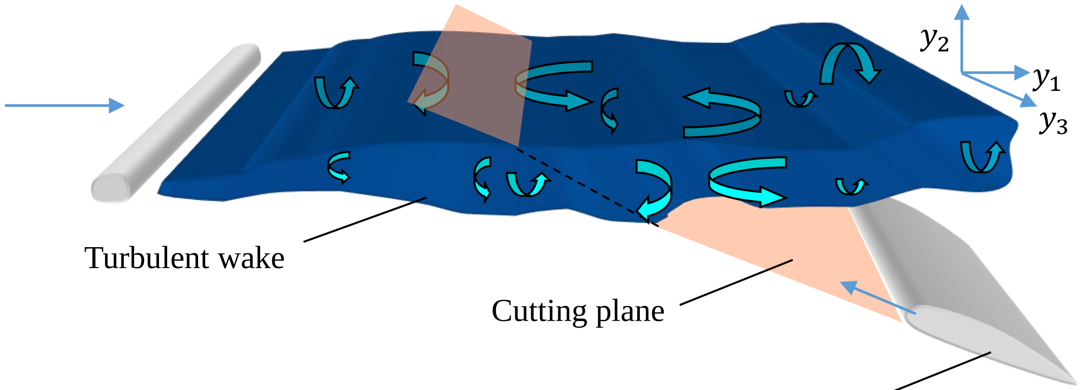
To characterize the typical behavior of a turbulent flow and the sound it produces we need to average. Averaging in time or space is not general enough. Thus we introduce the *expected value* $E[\,]$.

E.g. expected value of the velocity field generated by the wake cutting airfoil:

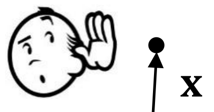
$$E[\mathbf{v}(\mathbf{y},t)] = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\mathbf{v}^{(n)}(\mathbf{y},t)$$

n^{th} realization of the turbulent velocity field





Expected values for aeroacoustic sources



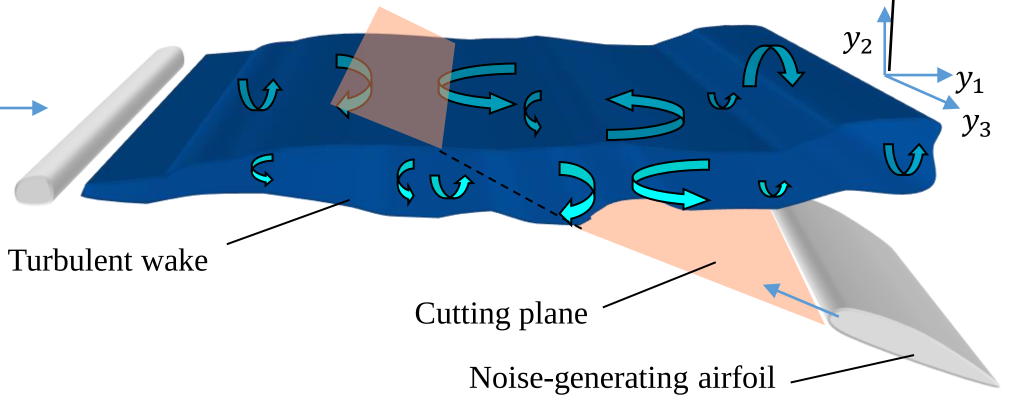
These are, in general, two-point averages. E.g. consider the far-field sound radiated due to the pressures generated by the turbulence on the airfoil. From Curle's equation we have the form:

$$p'(\mathbf{x},t) = \cdots \int_T \int_S p'(\mathbf{y},\tau)\{\dots\}\, dS(\mathbf{y})d\tau$$

To get the typical sound level at $\mathbf{x}$ we would take the mean square of $p'(\mathbf{x},t)$, i.e. $\overline{p'(\mathbf{x},t)^2} = \mathrm{E}[p'(\mathbf{x},t)^2]$

$$\overline{p'(\mathbf{x},t)^2} = \cdots E\left[\int_T \int_S p'(\mathbf{y},\tau)\{\dots\}dS(\mathbf{y})d\tau \int_T \int_S p'(\mathbf{y},\tau)\{\dots\}dS(\mathbf{y})d\tau\right]$$

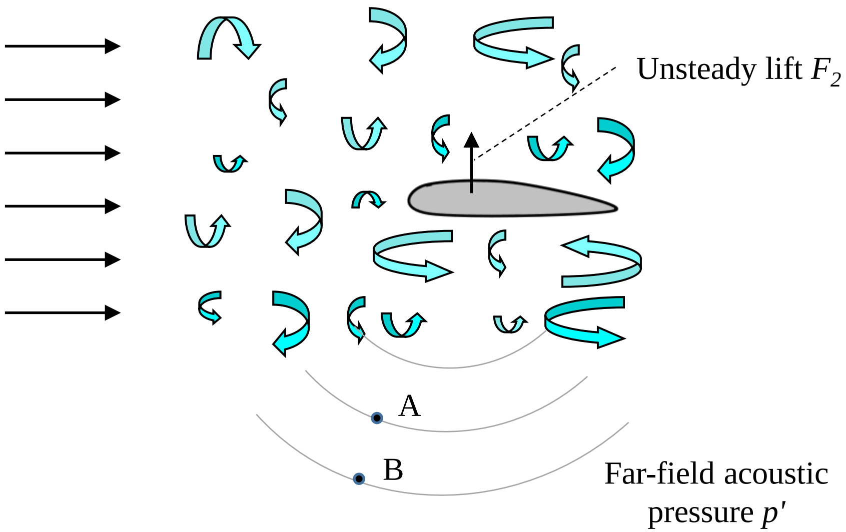
$$= \cdots \int_T \int_S \int_T \int_S E[p'(\mathbf{y},\tau)p'(\mathbf{y}',\tau')]\{\dots\}\{\dots\}' dS(\mathbf{y})d\tau\, dS(\mathbf{y}')d\tau'$$



2. Time spectra and correlations

Section 10.2

Consider a statistically stationary sound or flow variable that is varying stochastically.



How can we characterize its typical frequency content?

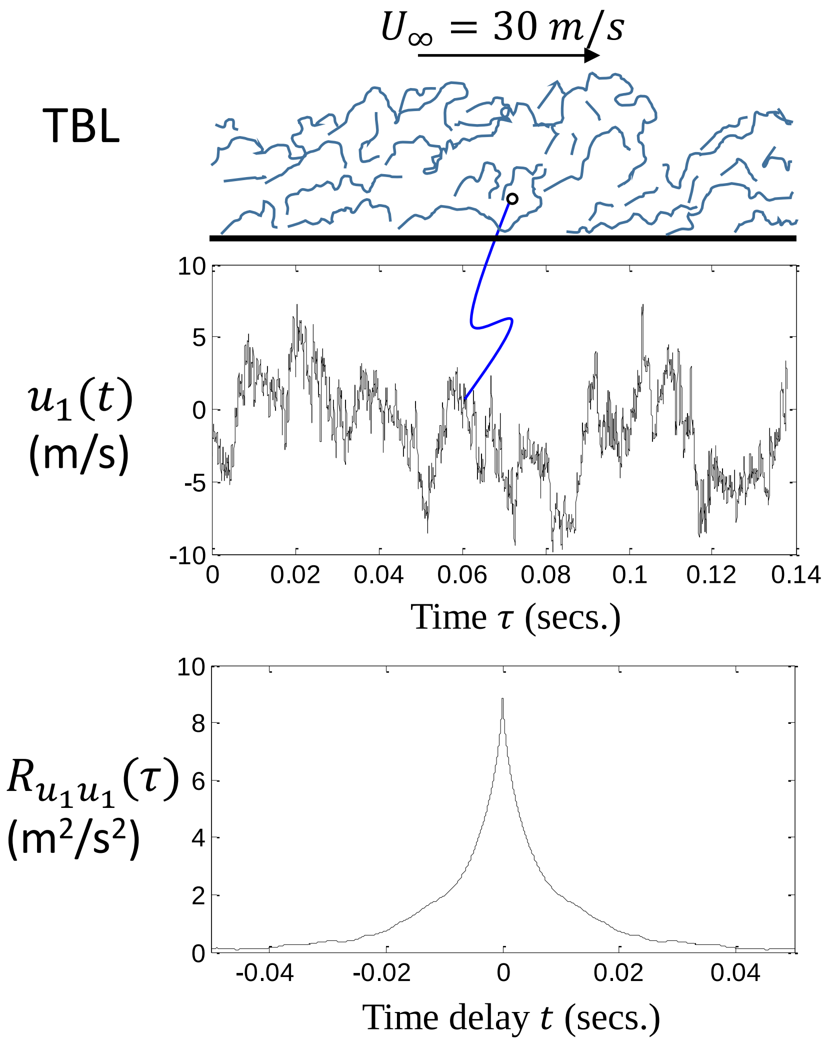
The Fourier transform? $$\tilde{a}(\omega) = \frac{1}{2\pi}\int_{-T}^{T} a(t)e^{i\omega t}dt$$

The spectrum ? $$S_{aa}(\omega) \equiv \frac{1}{2\pi}\int_{-T}^{T} R_{aa}(\tau)e^{i\omega\tau}d\tau$$

Understanding the correlation

$$\boxed{R_{aa}(\tau) \equiv E[a(t)a(t+\tau)]}$$

- Only a function of τ if time stationary
- $R_{aa}(0) = \overline{a^2}$
- Correlation coefficient function $\rho_{aa}(\tau) \equiv R_{aa}(\tau)/\overline{a^2}$
- Symmetric since $E[a(t)a(t+\tau)] = E[a(t)a(t-\tau)]$



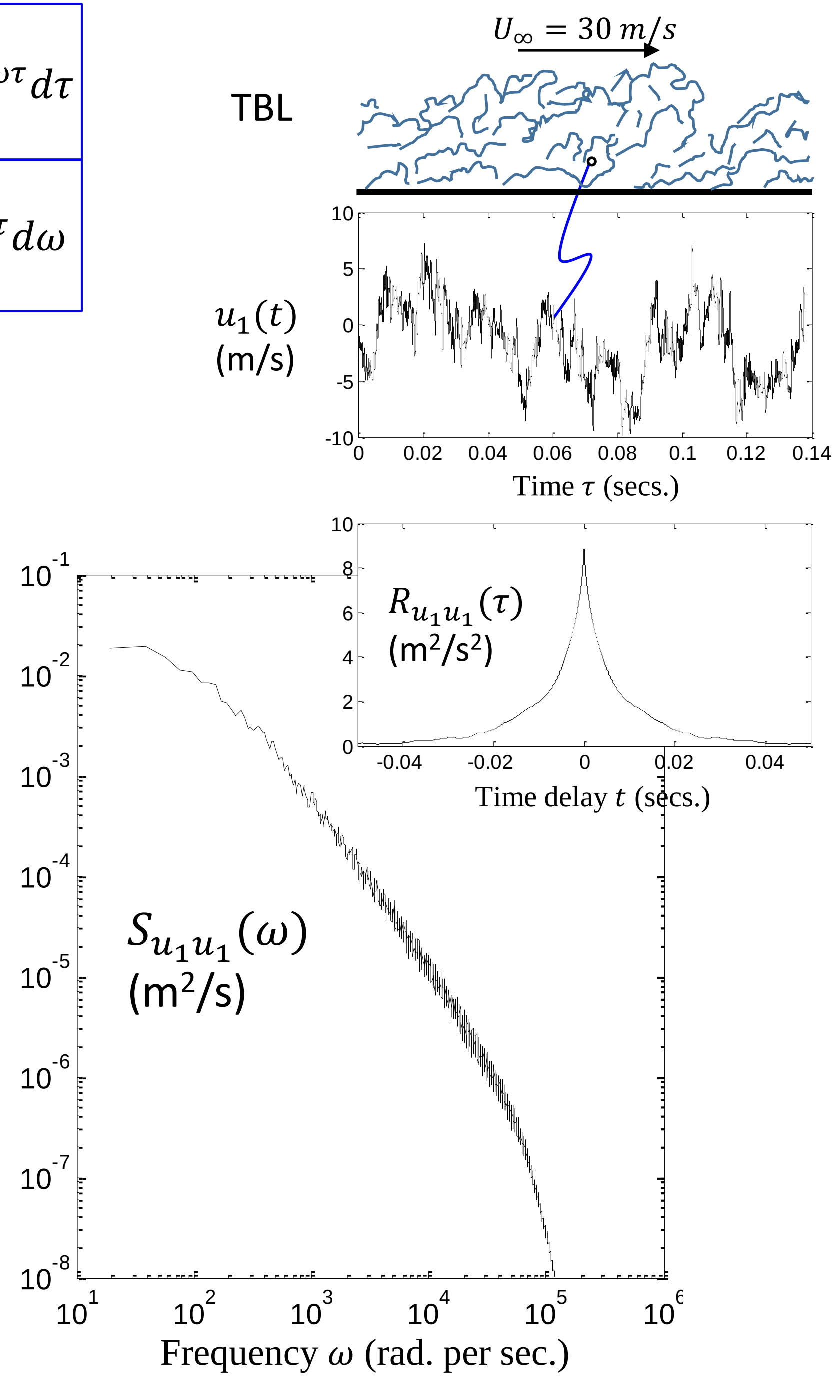
$\mathcal{T} = 0.0064\ s$ in this case. If mean velocity is 20m/s, approx. streamwise scale of turbulence can be estimated as $20 \times 0.0064 = 0.128\ m$

Understanding the spectrum

$$S_{aa}(\omega) \equiv \frac{1}{2\pi}\int_{-T}^{T} R_{aa}(\tau)e^{i\omega\tau}d\tau$$

$$R_{aa}(\tau) \equiv \int_{-\infty}^{\infty} S_{aa}(\omega)e^{-i\omega\tau}d\omega$$

- Separates the fluctuating quantity by scale
- Units are $\overline{a^2}$ per (radian per sec)
- At zero frequency $S_{aa}(0) = \frac{1}{2\pi}\int_{-T}^{T} R_{aa}(\tau)d\tau = \mathcal{T}\overline{a^2}/\pi$
- $S_{aa}(0)$ is real since R_{aa} is symmetric and exists for both positive and negative frequencies. Physically there is no difference.



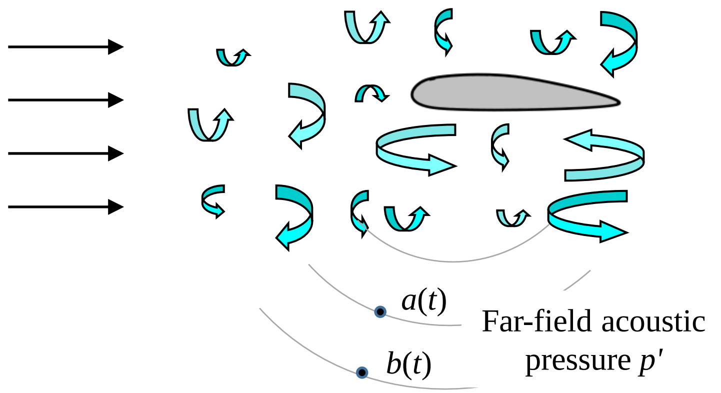
The spectrum and the Fourier transform

It is possible to show (see section 10.2) that there is a direct connection between the Fourier transform of a stochastic signal and its spectrum. Specifically, that,

This expected value approach is most often used when computing spectra from measured data. Note, however, the restriction on ω, and the fact that this method can never give the value of $S_{aa}(\omega)$ at zero frequency (which would give the integral timescale). The best we can do here is assume that the integral scale is given by the low frequency asymptote of the spectrum.

3. Cross correlations and spectra

Section 10.3



Consider two stochastic time-stationary variables $a(t)$ and $b(t)$ we define,

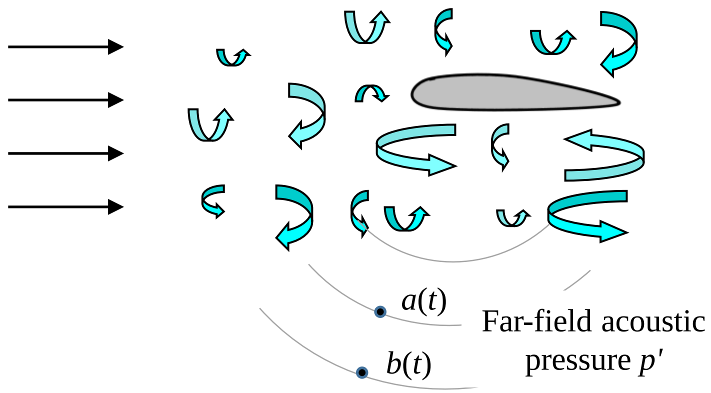
Cross correlation $$R_{ab}(\tau) \equiv E[a(t)b(t+\tau)]$$

Cross correlation coefficient $$\rho_{ab}(\tau) \equiv \frac{R_{ab}(\tau)}{\sqrt{\overline{a^2}\;\overline{b^2}}}$$

Cross spectral density $$S_{ab}(\omega) \equiv \frac{1}{2\pi}\int_{-T}^{T} R_{ab}(\tau)e^{i\omega\tau}d\tau$$

$$R_{ab}(\tau) = \int_{-\infty}^{\infty} S_{ab}(\omega)e^{-i\omega\tau}d\omega$$

Cross correlations and spectra



Consider two stochastic time-stationary variables $a(t)$ and $b(t)$ we define,

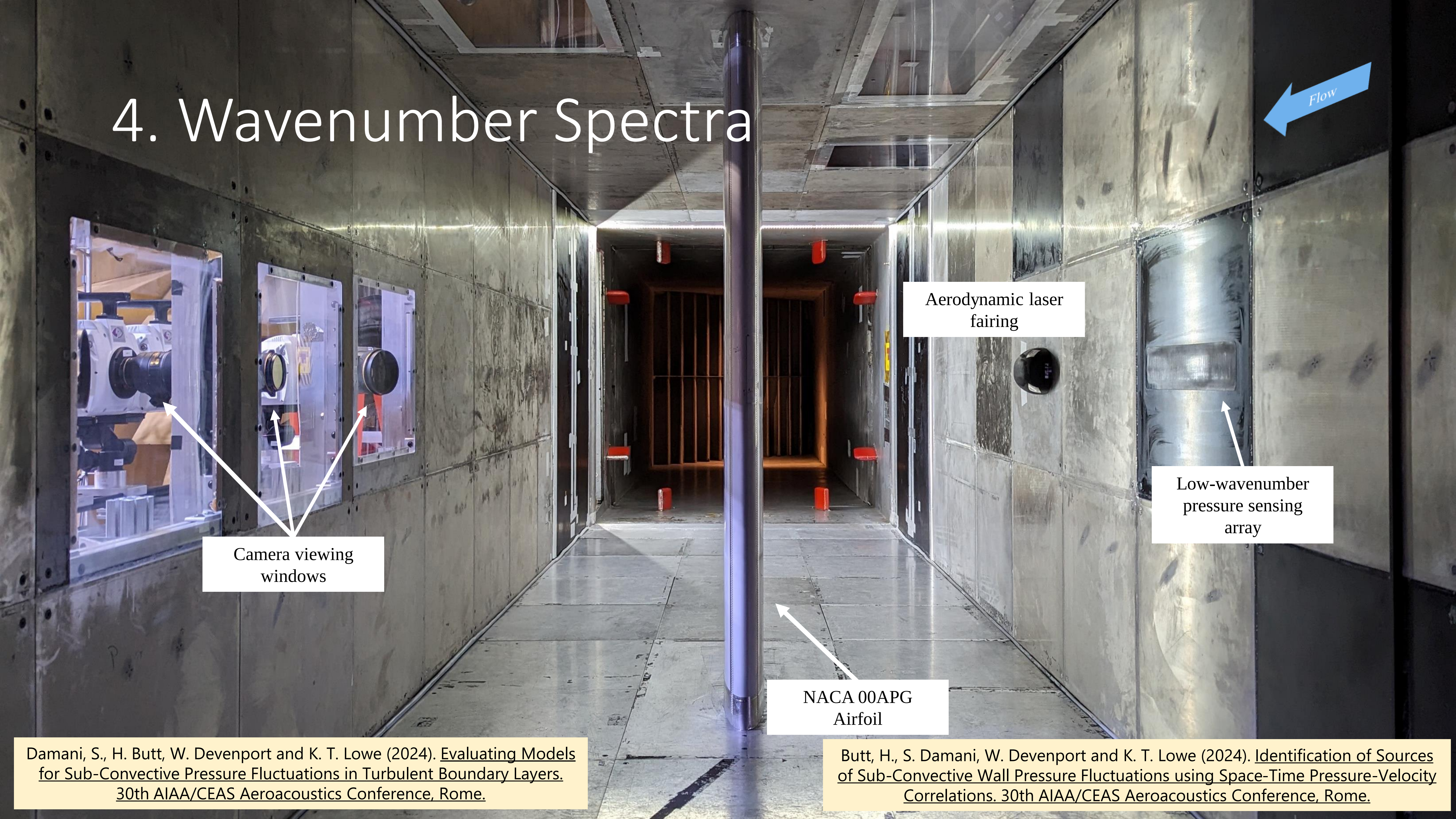
Coherence $\gamma_{ab}^2(\omega) \equiv \frac{\left|S_{ab}(\omega)^2\right|}{S_{aa}(\omega)S_{bb}(\omega)}$

Phase $\theta_{ab}(\omega) \equiv \arctan\left(\frac{Q_{ab}(\omega)}{C_{ab}(\omega)}\right)$

Analogously with the autospectrum we can show that

$$S_{ab}(\omega) = \frac{\pi}{T} E\left[\tilde{a}^*(\omega)\tilde{b}(\omega)\right] \qquad \omega \gg \pi/T$$

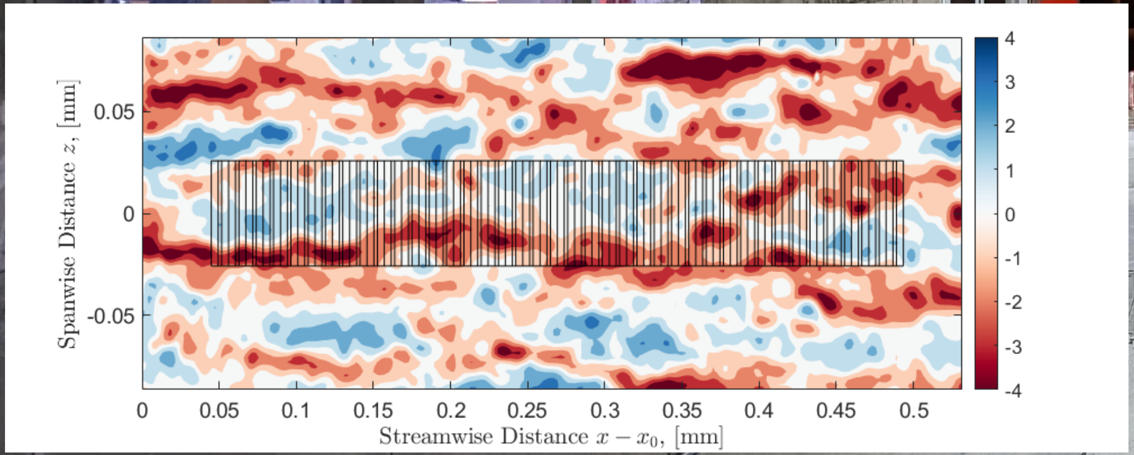
4. Wavenumber Spectra



Damani, S., H. Butt, W. Devenport and K. T. Lowe (2024). Evaluating Models for Sub-Convective Pressure Fluctuations in Turbulent Boundary Layers. 30th AIAA/CEAS Aeroacoustics Conference, Rome.

Butt, H., S. Damani, W. Devenport and K. T. Lowe (2024). Identification of Sources of Sub-Convective Wall Pressure Fluctuations using Space-Time Pressure-Velocity Correlations. 30th AIAA/CEAS Aeroacoustics Conference, Rome.

4. Wavenumber Spectra



Damani, S., H. Butt, W. Devenport and K. T. Lowe (2024). Evaluating Models for Sub-Convective Pressure Fluctuations in Turbulent Boundary Layers. 30th AIAA/CEAS Aeroacoustics Conference, Rome.

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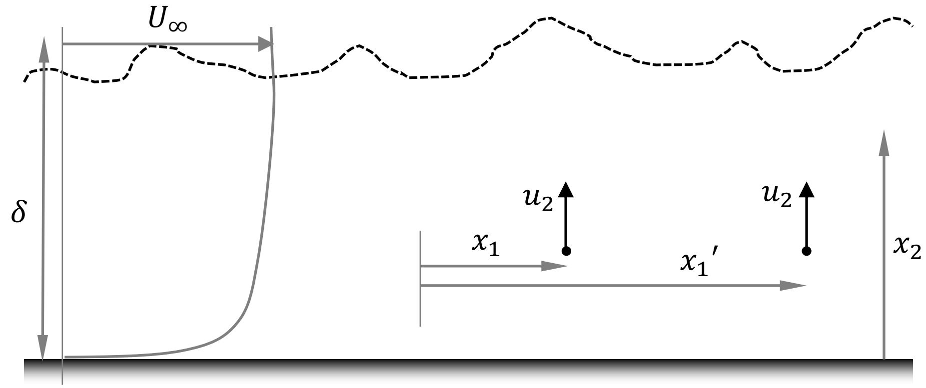
4. Wavenumber spectra

Section 10.4



Equivalent tools exist for describing the behavior of stochastic variables in space. Consider the vertical velocity fluctuation in a TBL. If the turbulence is homogeneous parallel to the wall, then we define $R_{22}(\Delta x_1) \equiv E[u_2(x_1)u_2(x_1')]$ where $\Delta x_1 = x_1' - x_1$

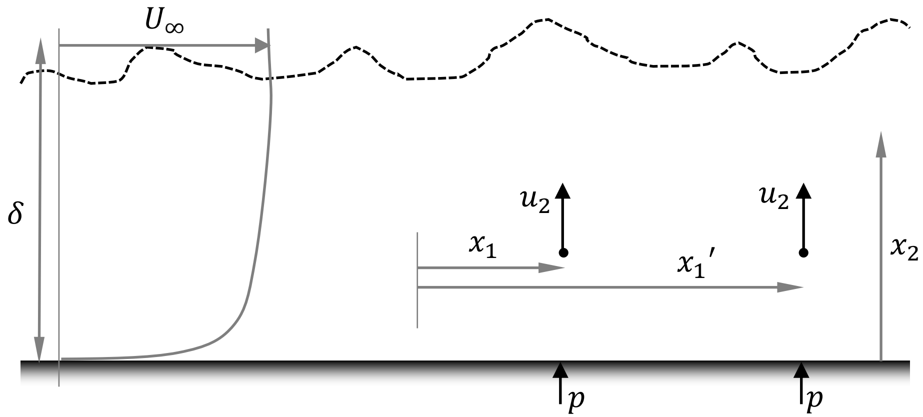
We can then define an integral lengthscale $L_{21} = \frac{1}{\overline{u_2^2}} \int_0^\infty R_{22}(\Delta x_1) d\Delta x_1$

…and a wavenumber spectrum $\phi_{22}(k_1) = \frac{1}{2\pi} \int_{-R\infty}^{R\infty} R_{22}(\Delta x_1) e_1^{-ik_1 \Delta x_1} d\Delta x_1$

…which is inverted as $R_{22}(\Delta x_1) = \int_{-\infty}^{\infty} \phi_{22}(k_1) e_1^{ik_1 \Delta x_1} dk_1$

These measures do not work well directions that are not homogeneous, e.g. x_2

Generalizing



In this boundary layer we could calculate the correlation between any two points and any two components at any two times, i.e. $E\left[u_i(x_1,x_2,x_3,t)u_j(x_1',x_2',x_3',t')\right] = R_{ij}(\Delta x_1, x_2, x_2', \Delta x_3, \tau)$

At any point we could calculate 6 integral scales, L_{i1} and L_{i3}

We could compute the 2-wavenumber spectrum in x_1 and x_3…

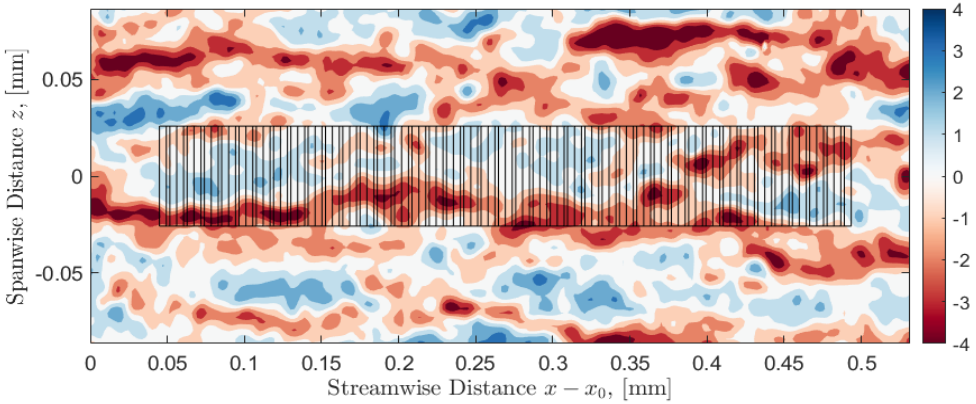
$$\phi_{ij}(x_2,x_2',k_1,k_3,\tau) = \frac{1}{(2\pi)^2}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty} R_{ij}(\Delta x_1, x_2, x_2', \Delta x_3, \tau) e_1^{-ik_1\Delta x_1 - ik_3\Delta x_3} d\Delta x_1\, d\Delta x_3$$

…and then Fourier transform in time to get the wavenumber frequency spectrum

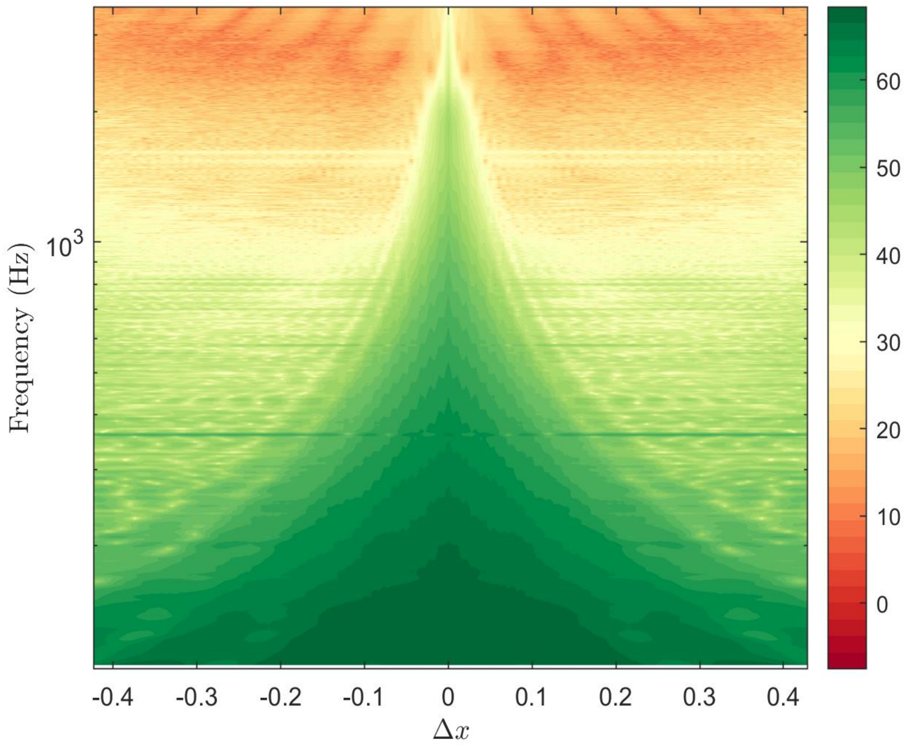
$$\Phi_{ij}(x_2,x_2',k_1,k_3,\omega) = \frac{1}{(2\pi)^3}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty} R_{ij}(\Delta x_1, x_2, x_2', \Delta x_3, \tau) e_1^{i\omega\tau-ik_1\Delta x_1 - ik_3\Delta x_3} d\Delta x_1\, d\Delta x_3\, d\tau$$

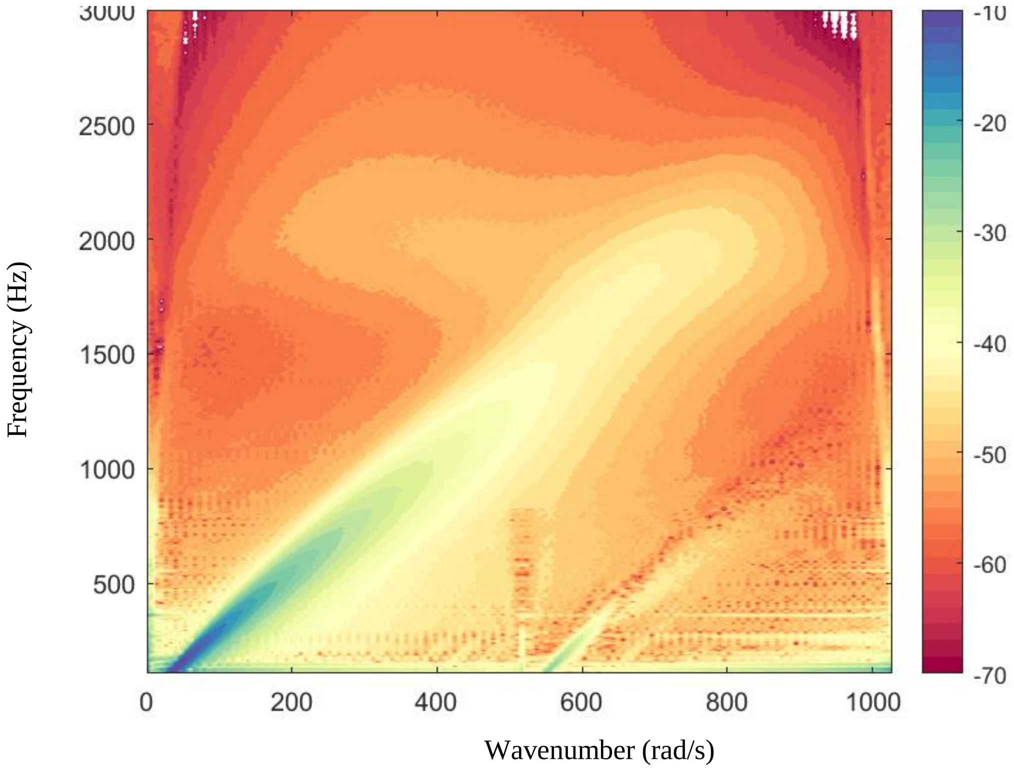
We can similarly write the wall pressure wave-number frequency spectrum as

$$\Phi_{pp}(k_1,k_3,\omega) = \frac{1}{(2\pi)^3}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty} R_{pp}(\Delta x_1, \Delta x_3, \tau) e_1^{i\omega\tau-ik_1\Delta x_1 - ik_3\Delta x_3} d\Delta x_1\, d\Delta x_3\, d\tau$$



U_e =23.62 m/s
Re_c= 1.2 million
δ= 67.5 mm
$\alpha = 0°$





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Butt, H., S. Damani, W. Devenport and K. T. Lowe (2024). Identification of Sources of Sub-Convective Wall Pressure Fluctuations using Space-Time Pressure-Velocity Correlations. 30th AIAA/CEAS Aeroacoustics Conference, Rome.

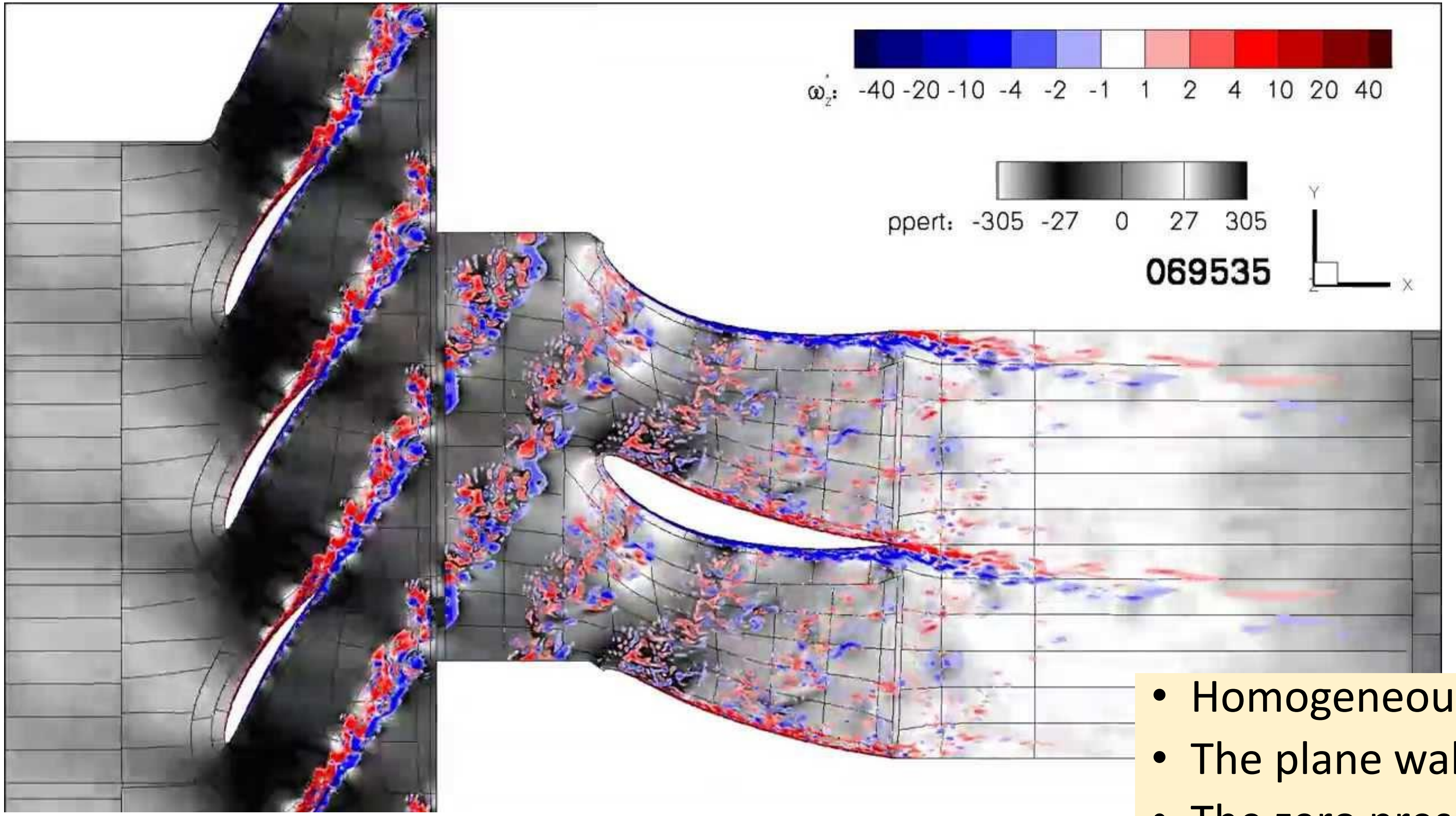
Summary, so far…

- The concept of the expected value $E[\,]$, the need for two-point statistics to define aeroacoustics sources
- The auto/cross correlation function and spectrum
- Integral time and length scales
- Coherence and phase
- Spatial correlations and wavenumber spectra
- Multi-dimensional correlations and spectra

$$\Phi_{ij}(x_2, x_2', k_1, k_3, \omega) = \frac{1}{(2\pi)^3} \int_{-R\infty}^{R\infty} \int_{-R\infty}^{R\infty} \int_{-R\infty}^{R\infty} R_{ij}(\Delta x_1, x_2, x_2', \Delta x_3, \tau) e_1^{i\omega\tau - ik_1\Delta x_1 - ik_3\Delta x_3} d\Delta x_1 \, d\Delta x_3 \, d\tau$$

5. Turbulence and aeroacoustics

https://www.youtube.com/channel/UCOfIMvwaq8pLodVcuoTdu4A/about



Greschner, B. and F. Thiele Broadband Noise Analysis of a Rotor-Stator-Cascade Using Wall Modeled LES Simulation. 18th AIAA/CEAS Aeroacoustics Conference (33rd AIAA Aeroacoustics Conference).

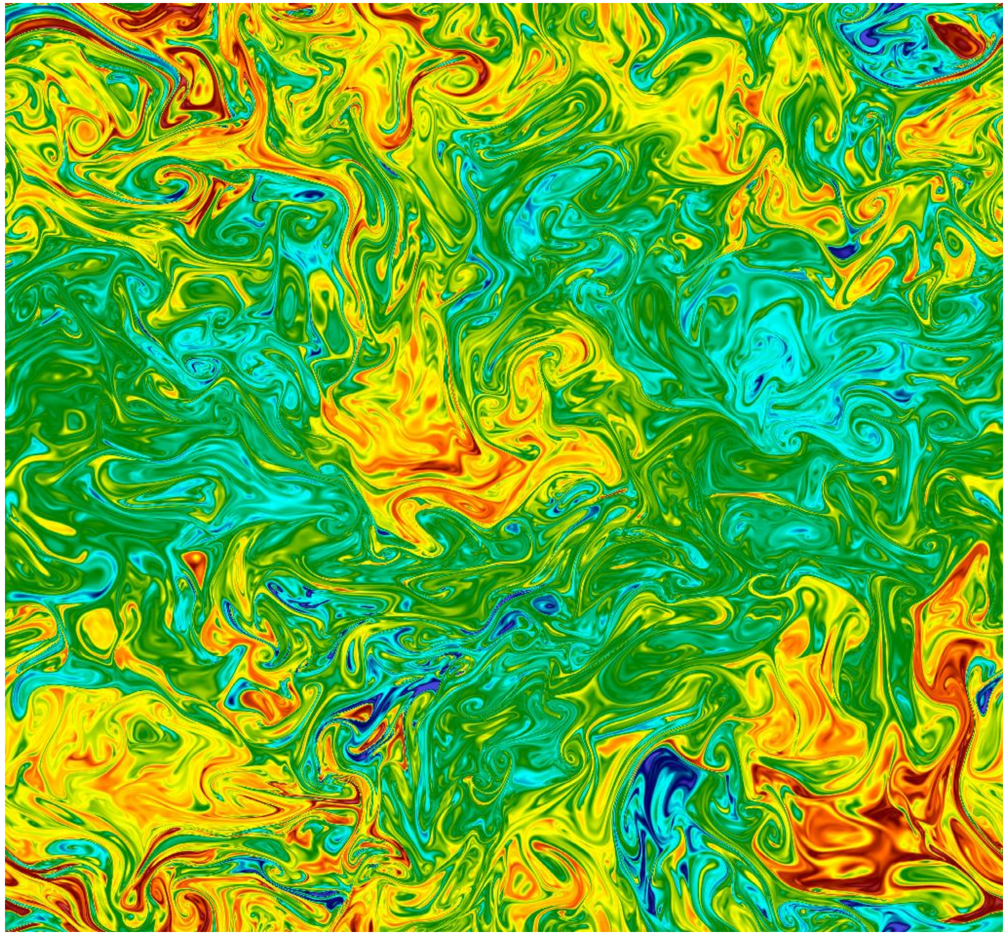
Ch 11 & 12

- Homogeneous turbulence
- The plane wake
- The zero pressure gradient turbulent boundary layer

6. Homogeneous isotropic turbulence

Section 11.3

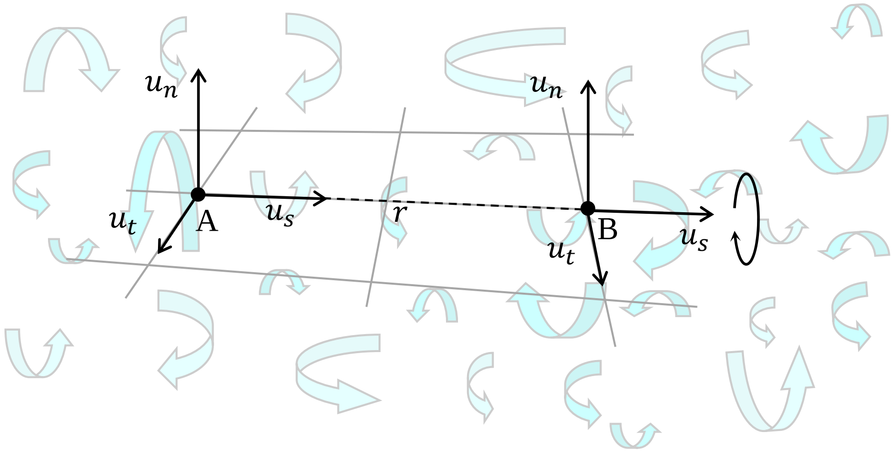
- Expected properties independent of position and direction
- Uniform/zero mean flow, decaying, no configuration-specific features
- Possibly represents the smaller scale behavior of all turbulent flows at high Re



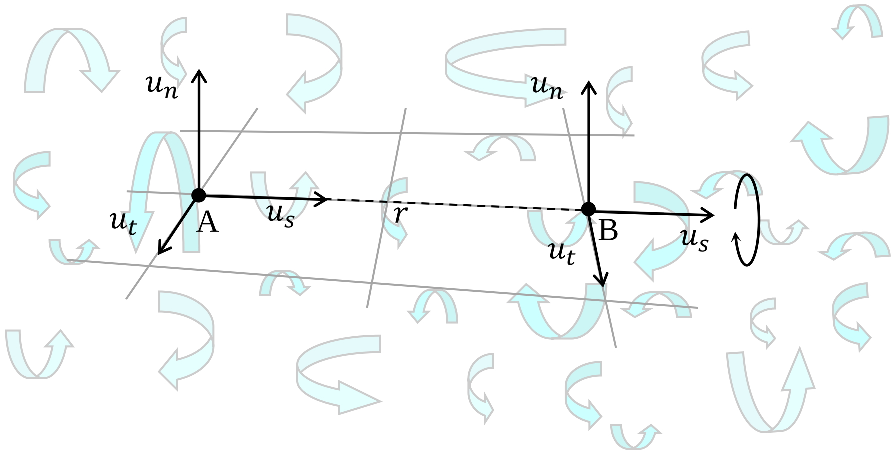
Lagaert, J. B., G. Balarac and G. H. Cottet (2014). "Hybrid spectral-particle method for the turbulent transport of a passive scalar." Journal of Computational Physics **260: 127-142.**

Single and two-point statistics

- We expect $R_{ij} = R_{ij}(\Delta x_1, \Delta x_2, \Delta x_3)$, but:
- Velocity fluctuations and lengthscales the same in all directions $\overline{u_1^2} = \overline{u_2^2} = \overline{u_3^2} = \overline{u^2}$
- Two point velocity correlations the same in all directions, so $\mathrm{E}[u_s(A)u_s(B)]$ is the same $f(r)$ regardless of the direction of r
- Same for $\mathrm{E}[u_n(A)u_n(B)] = E[u_t(A)u_t(B)]$ which are the same $g(r)$ regardless.
- This means that R_{ij} can be written in terms of only two scalar functions.
- These functions can be related to each other through continuity.

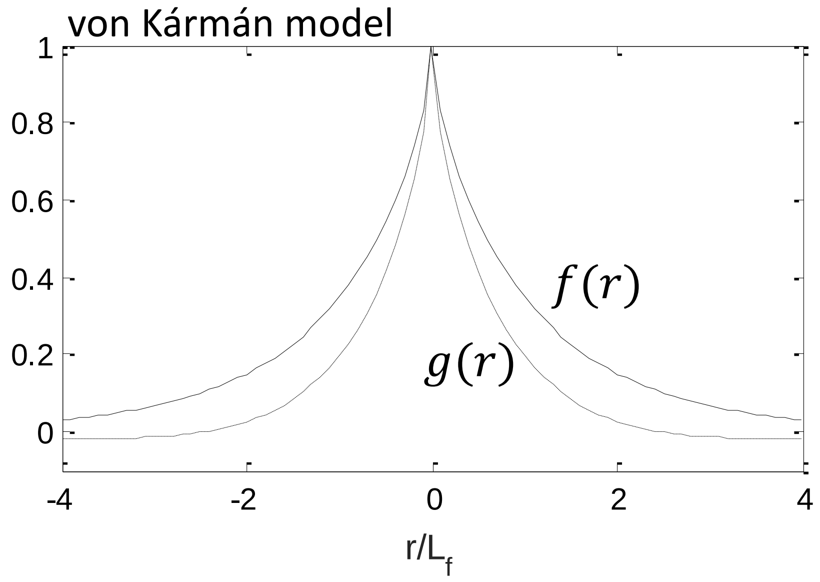


Correlation function



$$R_{ij}(\Delta x_1, \Delta x_2, \Delta x_3) = \overline{u^2}\left[g(r)\delta_{ij} + \frac{f(r) - g(r)}{r^2}\Delta x_i \Delta x_j\right]$$

$$g = f + \frac{r}{2}\frac{\partial f}{\partial r}$$



Note k is the wavenumber vector magnitude here (not acoustic wavenumber)

Wavenumber spectrum

$$\varphi_{ij}(k_1,k_2,k_3) \equiv \frac{1}{(2\pi)^3}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty}\int_{-R_\infty}^{R_\infty} R_{ij}(\Delta x_1,\Delta x_2,\Delta x_3)e^{-ik_1\Delta x_1-ik_2\Delta x_2-ik_3\Delta x_3}d\Delta x_1 d\Delta x_2 d\Delta x_3$$

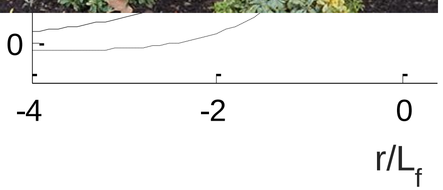
$$\boxed{\varphi_{ij}(k_1,k_2,k_3) = \frac{E(k)}{2\pi k^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)}$$

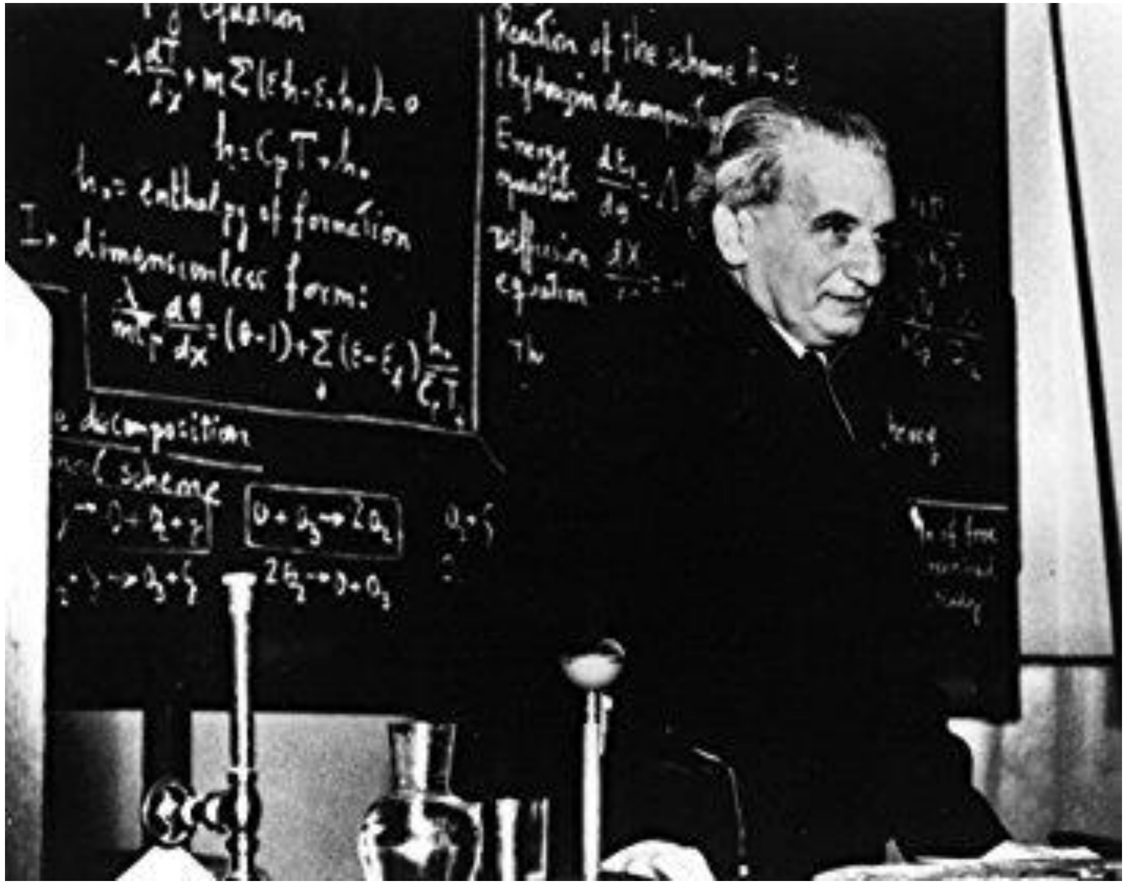
$$\boxed{R_{ij}(\Delta x_1,\Delta x_2,\Delta x_3) = \overline{u^2}\left[g(r)\delta_{ij}+\frac{f(r)-g(r)}{r^2}\Delta x_i\Delta x_j\right]}$$

Theodore von Kármán

May 11th 1881 to May 6th 1963

GALcit





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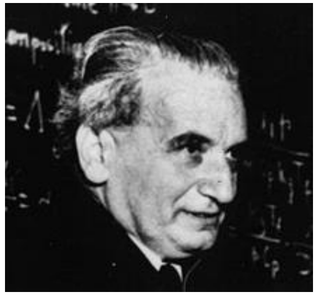
FOR FLUID DYNAMICS

The von Kármán spectrum

Section 11.3.2

von Kármán, T., "Progress in the Statistical Theory of Turbulence," Proceedings of the National Academy of Sciences 1948, vol. 34, no. 11, pp. 530-539.

https://en.wikipedia.org/wiki/Theodore_von_K%C3%A1rm%C3%A1n



Based upon theoretical considerations and experimental comparison von Kármán proposed the form

$$E(k) = \frac{55}{9\pi} L_f \frac{\overline{u^2}}{k_e} \frac{(k/k_e)^4}{[1+(k/k_e)^2]^{17/6}}$$

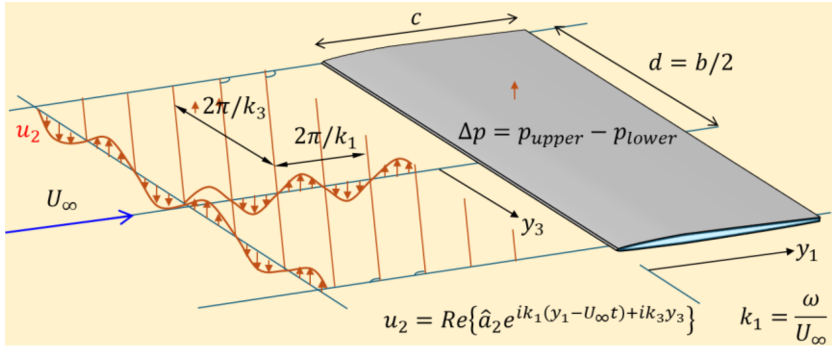
from which all other two-point properties can be derived

Examples…

The longitudinal and lateral correlation functions $f(r) = \frac{2^{2/3}}{\Gamma(^1/_3)} (k_e r)^{\frac{1}{3}} K_{1/3}(k_e r)$

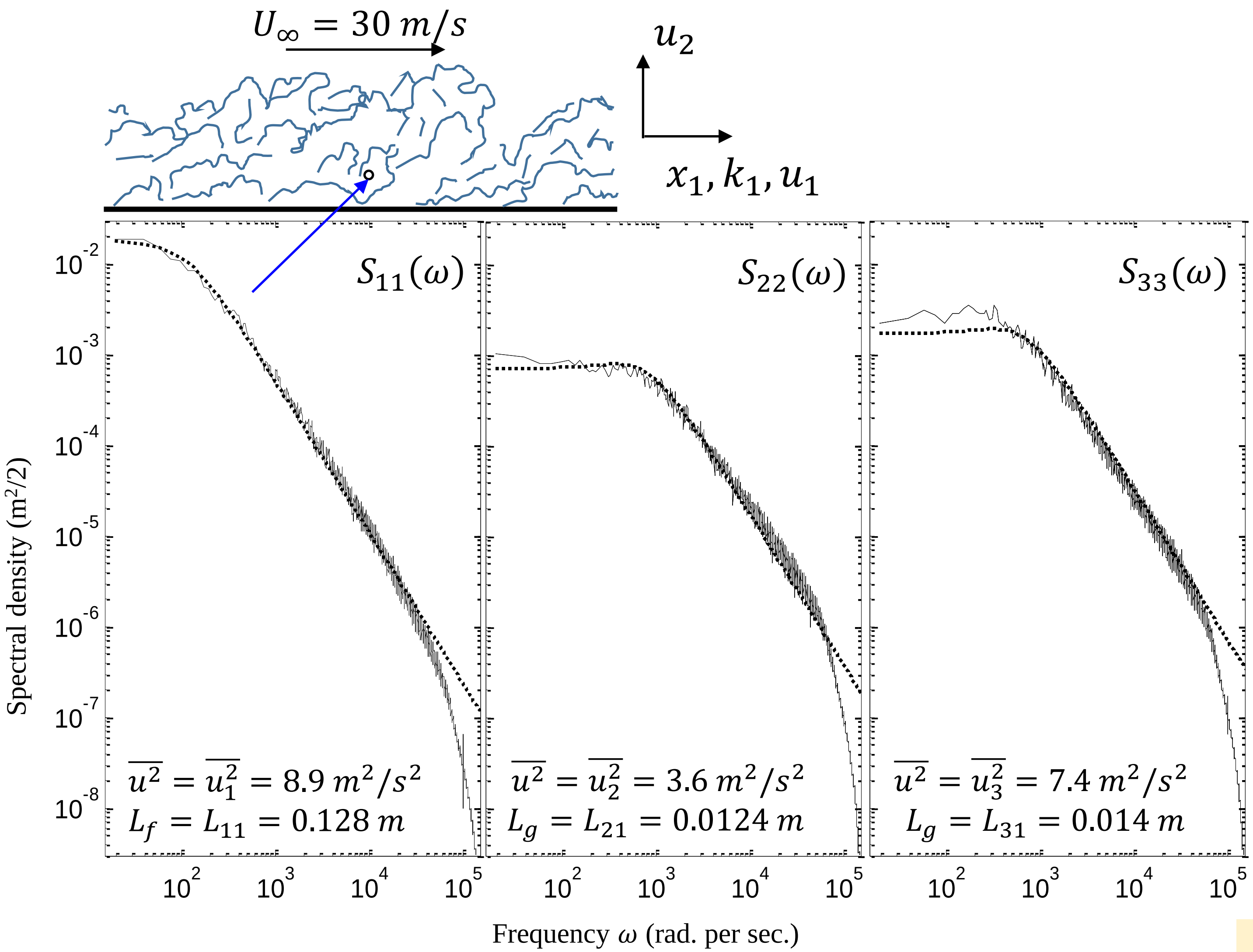
and $g(r) = \frac{2^{2/3}}{\Gamma(^1/_3)} (k_e r)^{\frac{1}{3}} \left(K_{1/3}(k_e r) - \frac{k_e r}{2} K_{-2/3}(k_e r) \right)$

The 1D longitudinal and lateral spectra along a line through the turbulence $\phi_{11}(k_1) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(^5/_6)}{\Gamma(^1/_3)} \frac{\overline{u^2}}{k_e} \frac{1}{[1+(k_1/k_e)^2]^{5/6}}$



The upwash wavenumber spectrum on a plane through the turbulence $\phi_{22}(k_1, k_3) = \frac{4}{9\pi} \frac{\overline{u^2}}{k_e^2} \frac{(k_1^2+k_3^2)/k_e^2}{[1+(k_1^2+k_3^2)/k_e^2]^{7/3}}$

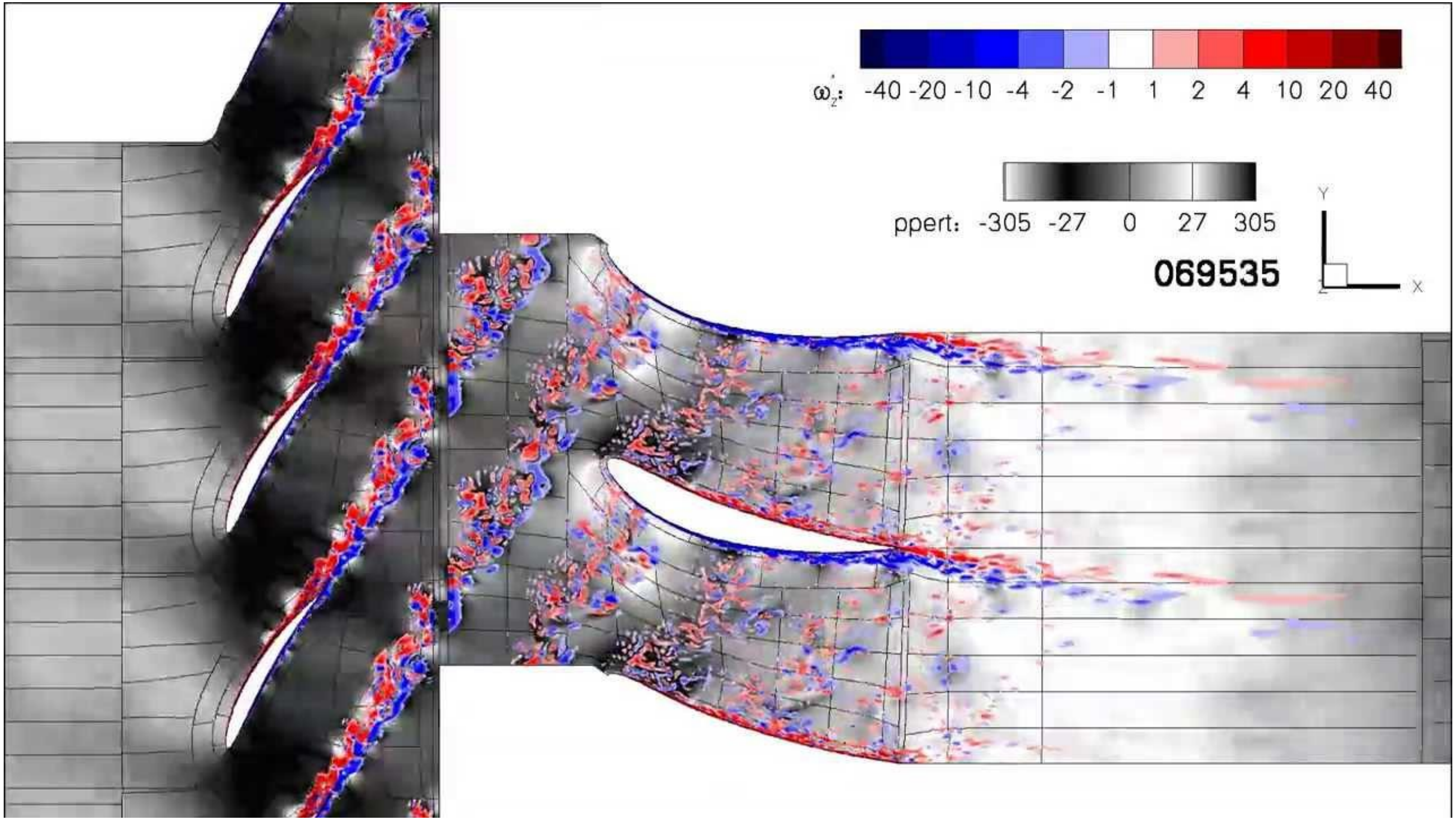
vs. TBL data



Other models:

- Leipmann (less accurate, algebraically simpler), 11.3.3
- Pope (includes dissipation region), 11.3.2
- Kerschen Gliebe (allows for anisotropy). 11.3.4
- Rapid distortion theory (can account for arbitrary rapid distortion), 11.6

Morton, M. A. (2012). MS Thesis, Virginia Tech, http://hdl.handle.net/10919/34467.

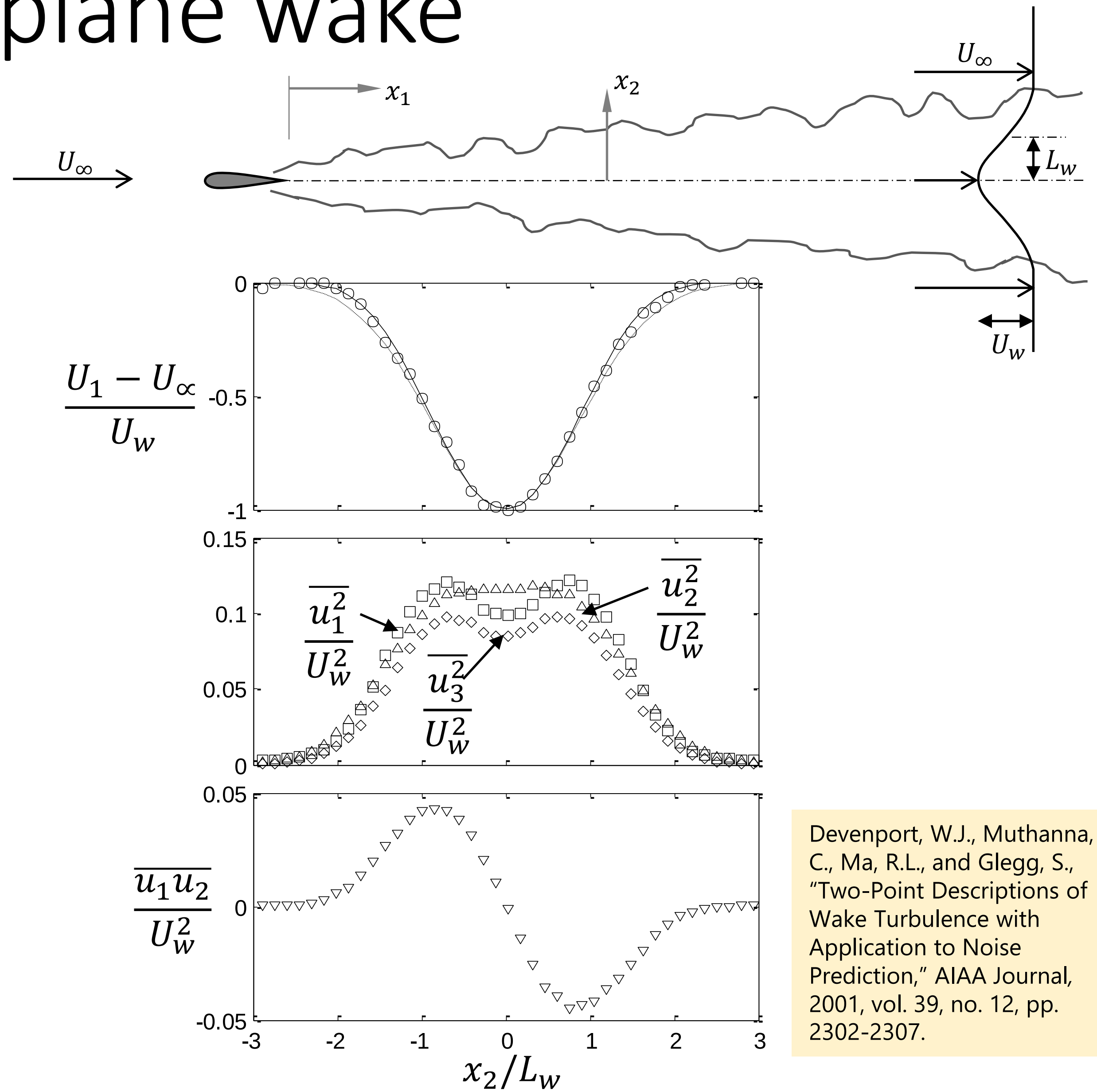


Rotor-Stator-Cascade: IDDES simulation of interaction noise, spanwise vorticity

Greschner, B. and F. Thiele Broadband Noise Analysis of a Rotor-Stator-Cascade Using Wall Modeled LES Simulation. 18th AIAA/CEAS Aeroacoustics Conference (33rd AIAA Aeroacoustics Conference).

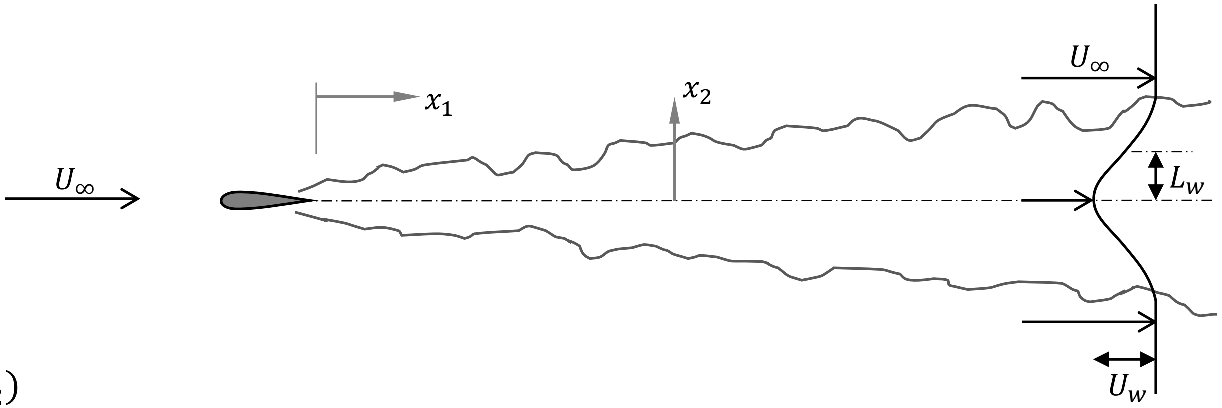
7. The fully developed plane wake

- Formed in the far-wake of 2D bodies.
- Slender flow $\partial/\partial x_1 \ll \partial/\partial x_2$
- Mean flow and turbulence structure become self-similar, i.e. constant when scaled on L_w and U_w

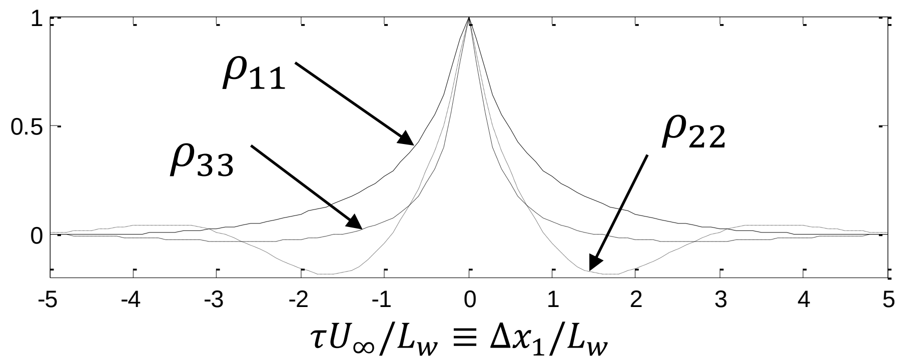


Devenport, W.J., Muthanna, C., Ma, R.L., and Glegg, S., "Two-Point Descriptions of Wake Turbulence with Application to Noise Prediction," AIAA Journal, 2001, vol. 39, no. 12, pp. 2302-2307.

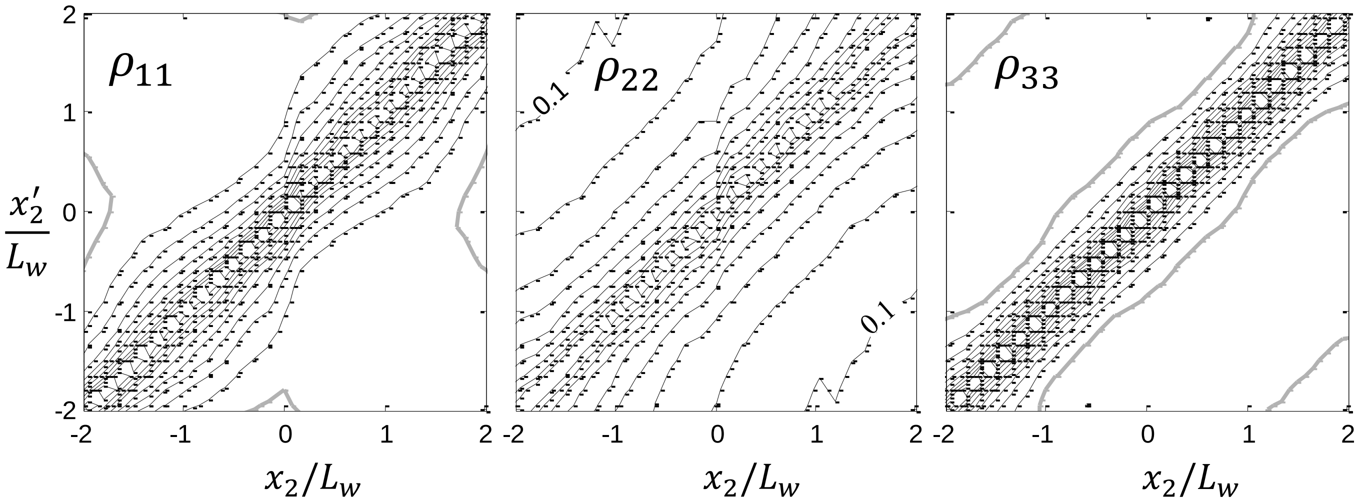
Two-point properties



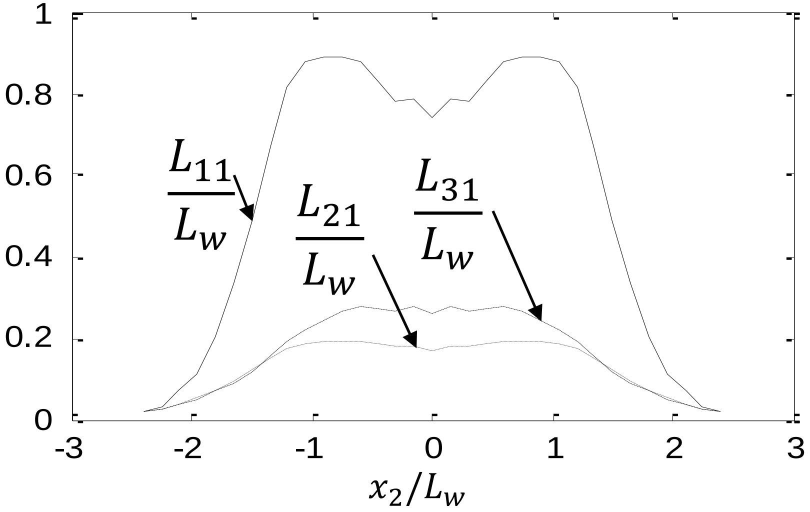
$\rho_{ij}(\Delta x_1)$ inferred from $\rho_{ij}(\tau)$ along wake centerline



$\rho_{ij}(x_2, x_2')$



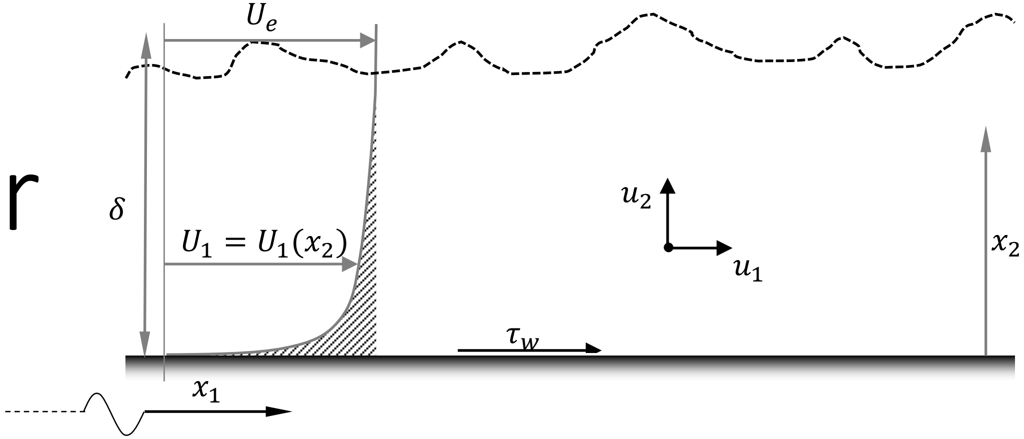
Streamwise integral scales



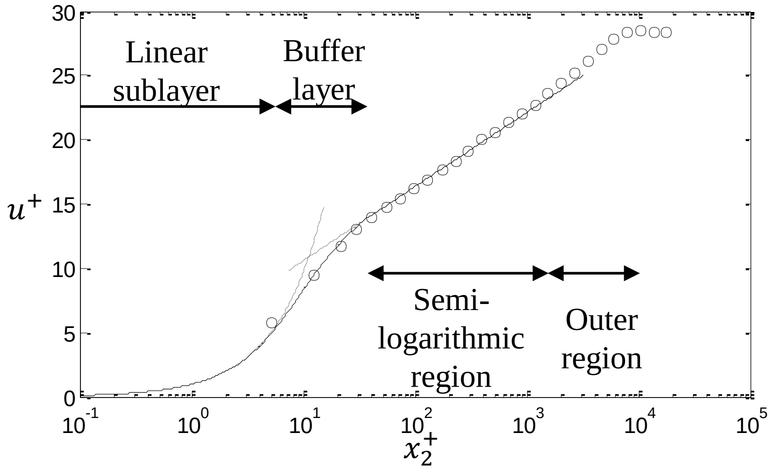
Glegg, S. A. L. and Devenport, W. J. (2001). "Proper Orthogonal Decomposition of Turbulent Flows for Aeroacoustic and Hydroacoustic Applications."Journal of Sound and Vibration **239(4): 767-784.**

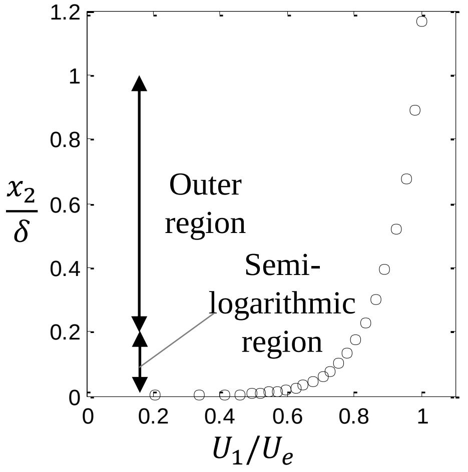
Data on www.aeroacoustics.net

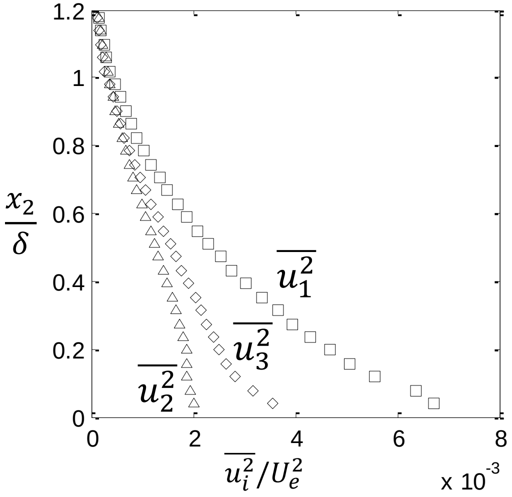
8. Flat plate boundary layer

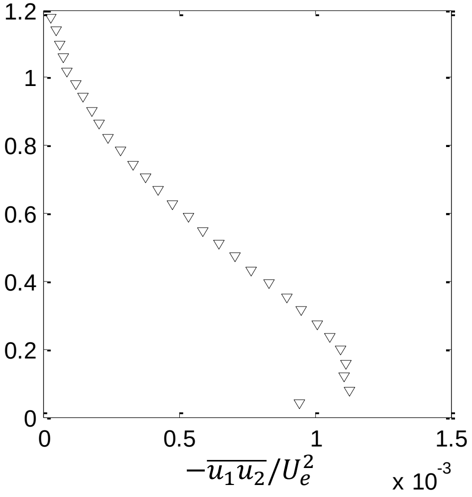


- Forms due to no slip condition.
- Slender flow $\partial/\partial x_1 \ll \partial/\partial x_2$
- Velocity scales
- Distance scales



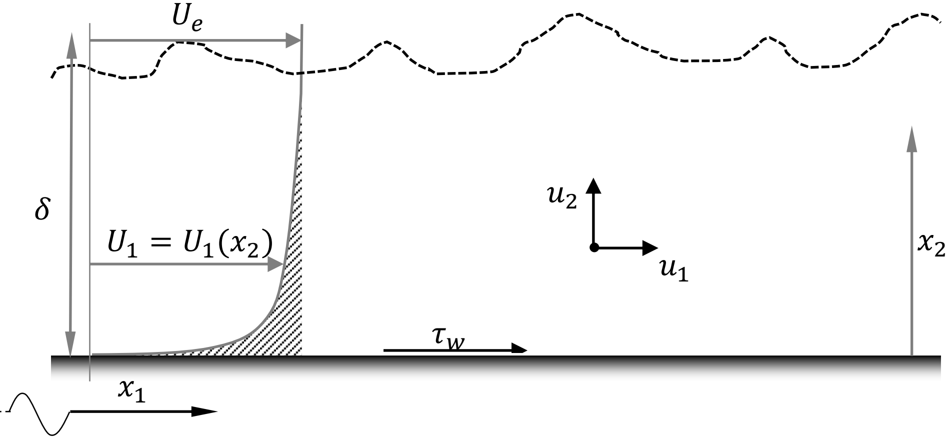






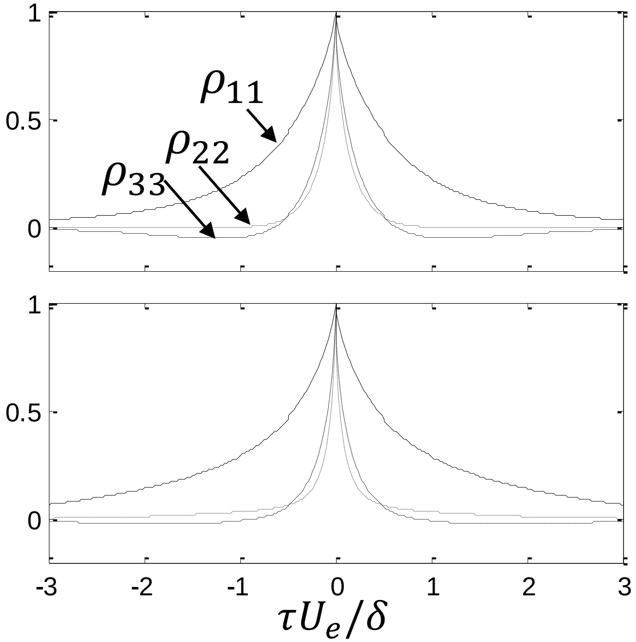
Morton, M. A., Devenport, W. J. and Glegg, S. (2012). Rotor Inflow Noise Caused by a Boundary Layer: Inflow Measurements and Noise Predictions. 18th AIAA/CEAS Aeroacoustics Conference. Colorado Springs, Colorado.

Two-point properties

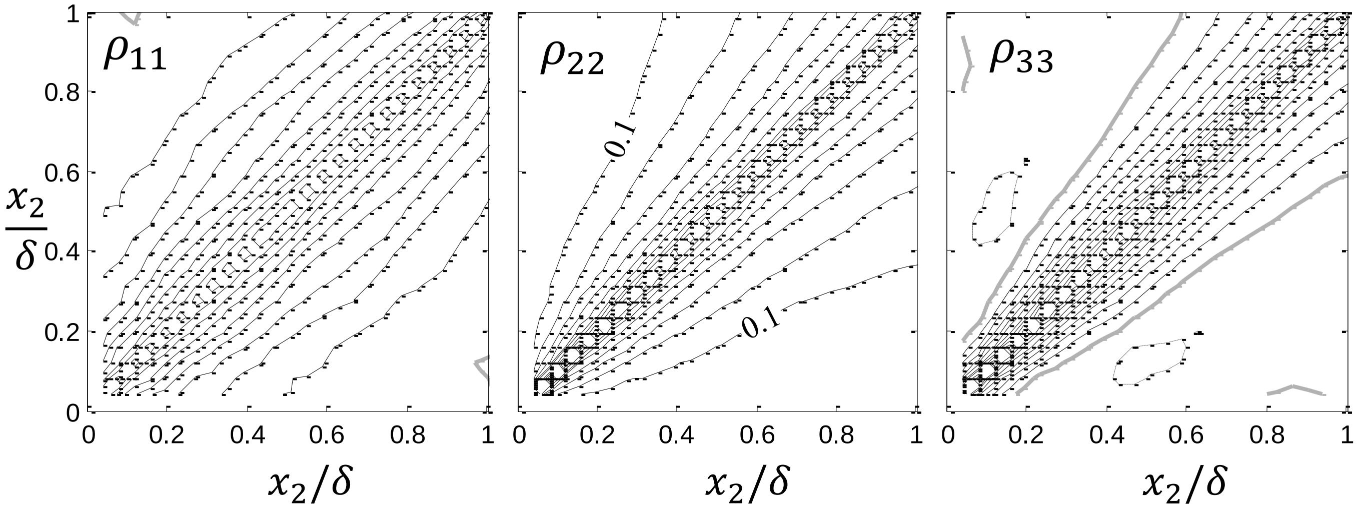


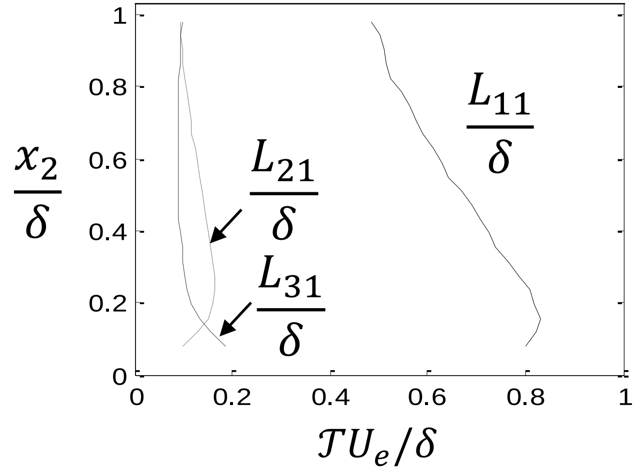
$\rho_{ij}(\tau)$ at $x_2/\delta = 0.51$

$\rho_{ij}(\tau)$ at $x_2/\delta = 0.20$



$\rho_{ij}(x_2, x_2')$

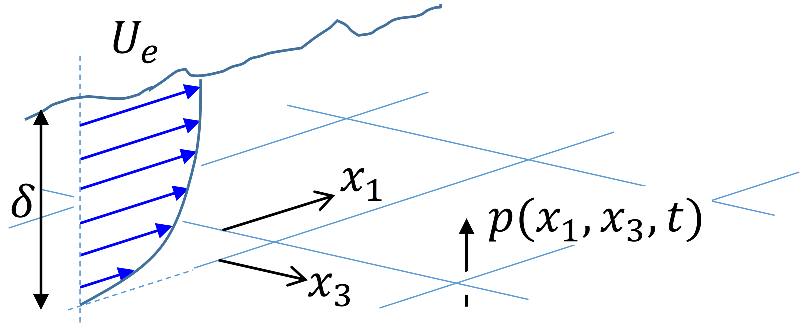




Morton, M. A. (2012). "Rotor Inflow Noise Caused by a Boundary Layer: Inflow Measurements and Noise Predictions." MS Thesis, Virginia Tech, http://hdl.handle.net/10919/34467.

Data on www.aeroacoustics.net

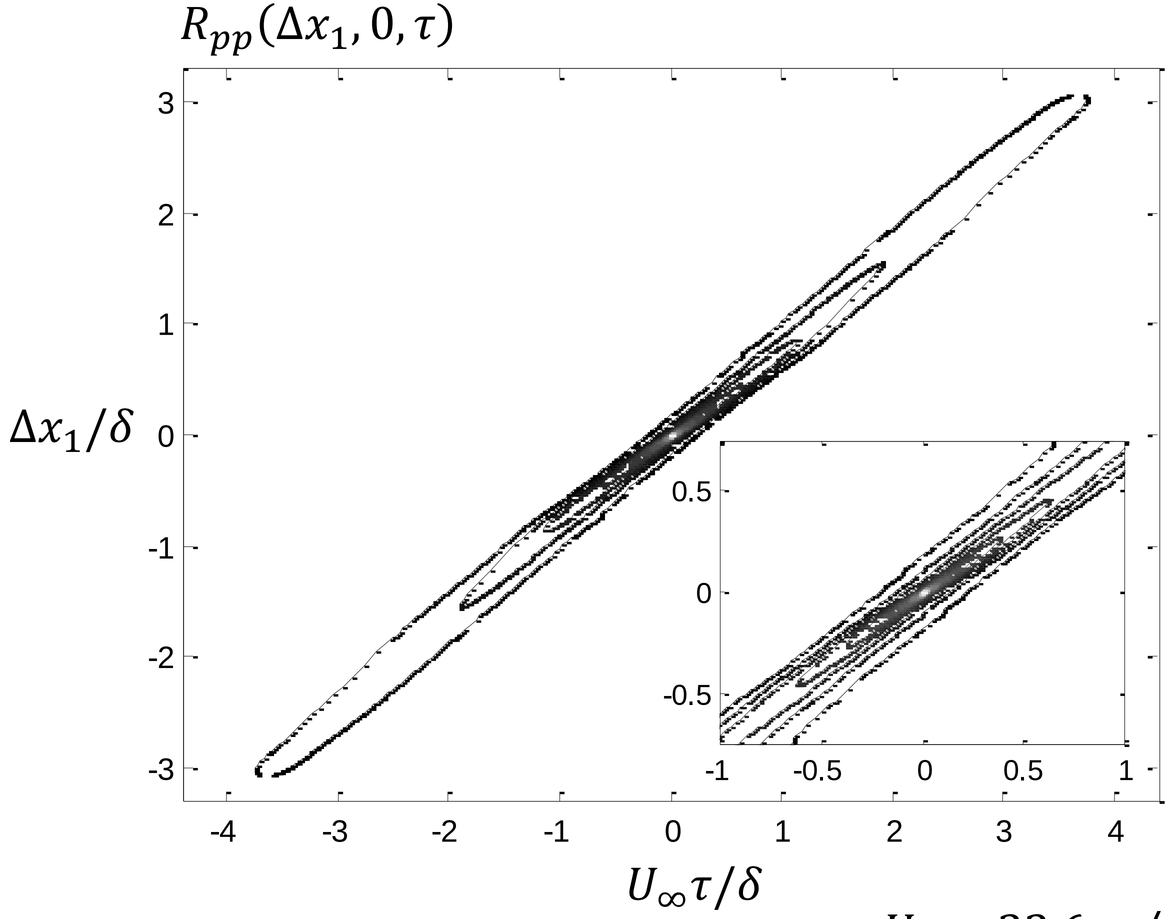
Wall pressure fluctuations

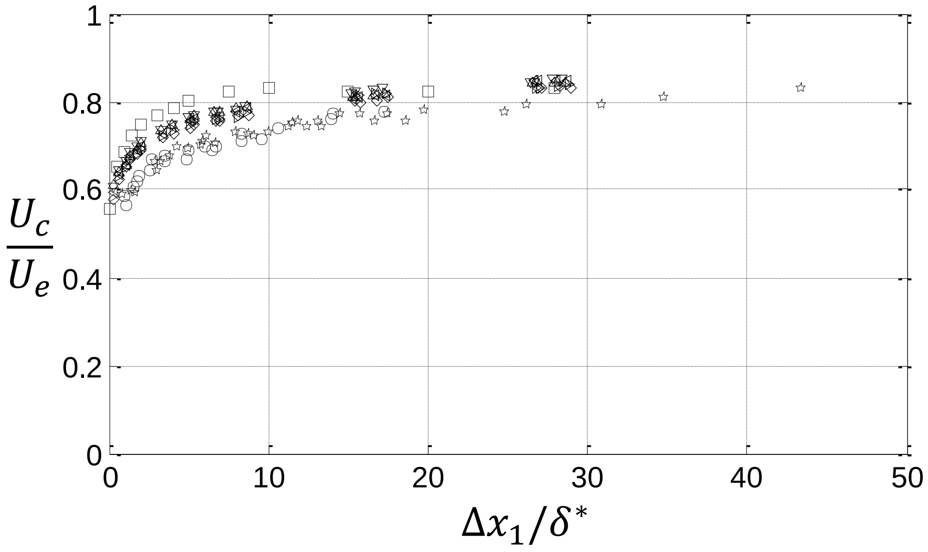


$$R_{pp}(\Delta x_1, \Delta x_3, \tau) = E[p(x_1, x_3, t)p(x_1', x_3', t')]$$

Correlation forms a strip (<u>convective ridge</u>) because the eddies producing the pressure fluctuations are swept along by the flow.

The big eddies live longer so correlate over a larger distance so this <u>convection velocity U_c</u> (reflected in the slope of the ridge increases with Δx_1

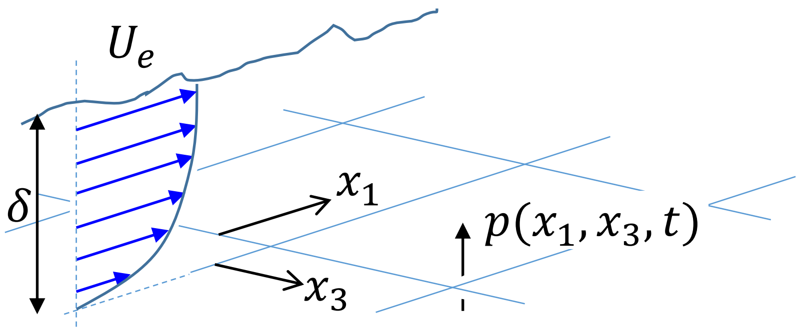




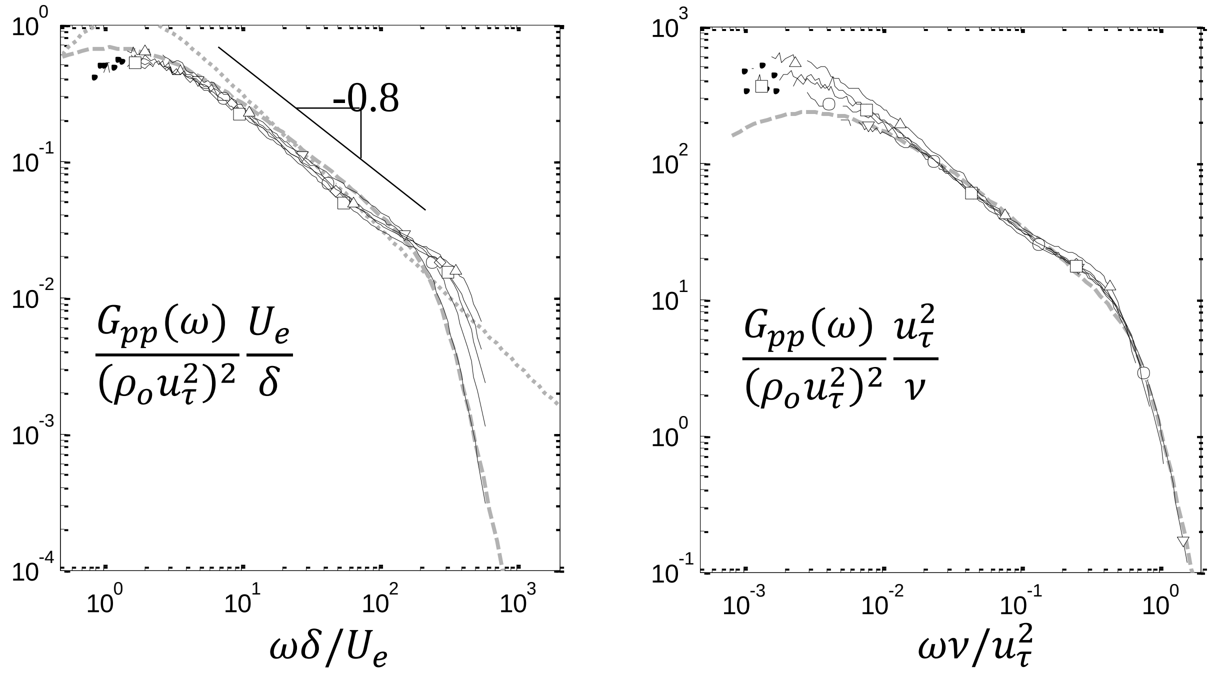
$U_e = 33.6\,m/s$
$\delta = 231\,mm$

Forest, J. B. (2012). The wall pressure spectrum of high Reynolds number rough-wall turbulent boundary layers. <u>Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University.</u> **<u>MS.</u>**

Pressure frequency spectrum



$$G_{pp}(\omega) = \frac{1}{\pi}\int_{-T}^{T} R_{pp}(\tau)e^{i\omega\tau}d\tau$$



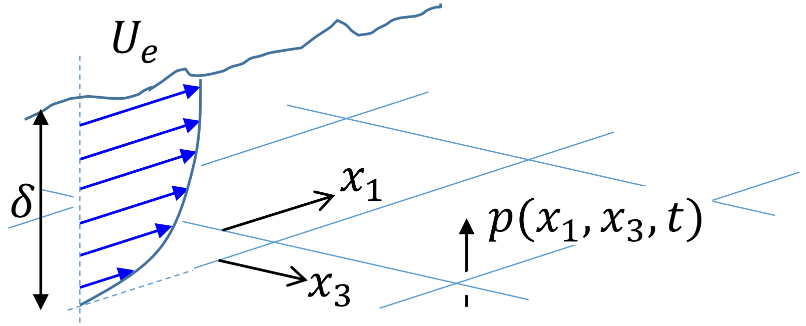
At low frequencies spectra tend to look the same when the frequency is scaled with δ and U_e (representing the large eddies of the outer region).

At high frequencies they look the same when frequency is normalized on the viscous frequency scale u_τ^2/ν (representing the near wall eddies).

In either region the pressure fluctuation amplitude scales on the wall shear stress τ_w

Meyers, T., Forest, J. B. and Devenport, W. J. (2015). "The wall-pressure spectrum of high-Reynolds-number turbulent boundary-layer flows over rough surfaces." Journal of Fluid Mechanics **768: 261-293.**

Pressure frequency spectrum



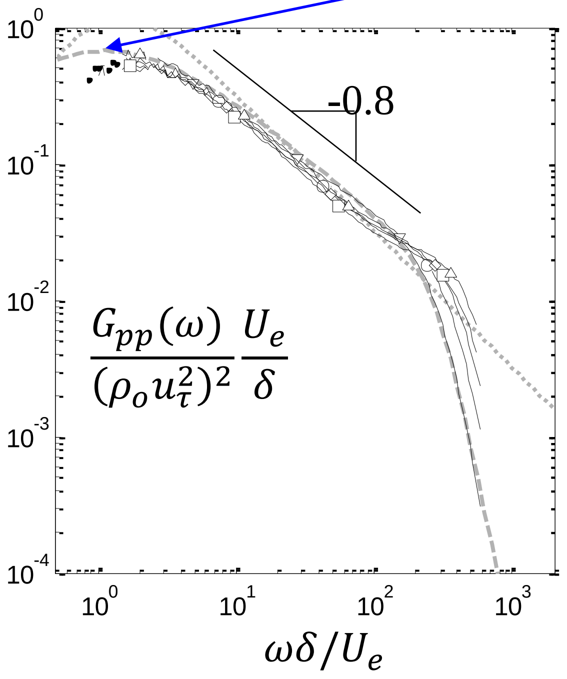
$$G_{pp}(\omega) = \frac{1}{\pi}\int_{-T}^{T} R_{pp}(\tau)e^{i\omega\tau}d\tau$$

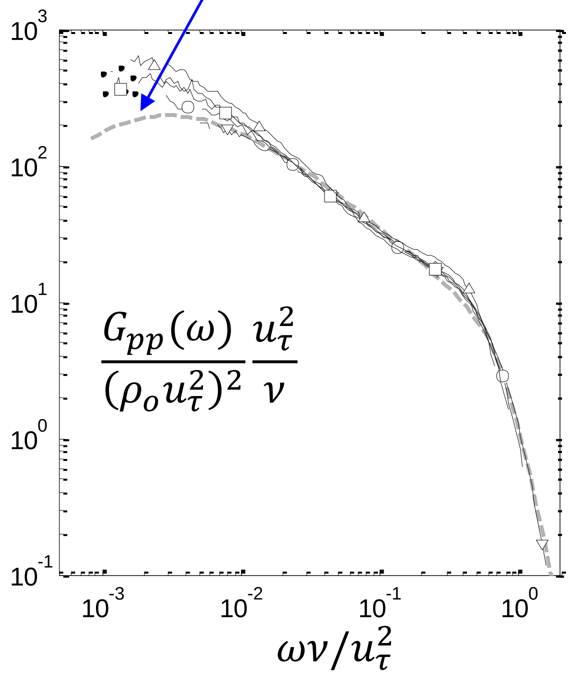
Goody model

Eq. 12.1.5

$$\frac{G_{pp}(\omega)}{(\rho_o u_\tau^2)^2}\frac{U_e}{\delta} = \frac{C_2(\omega\delta/U_e)^2}{[(\omega\delta/U_e)^n + C_1]^{3.7} + \left[C_3 R_T^{-4/7}(\omega\delta/U_e)\right]^7}$$

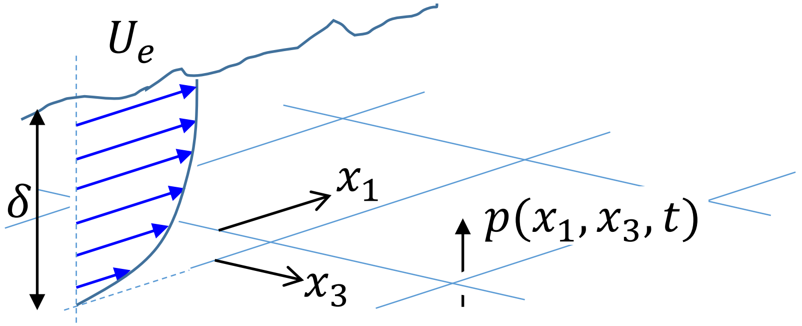
With $R_T = \frac{\delta}{U_e}\frac{\nu}{u_\tau^2}, C_1 = 0.5, C_2 = 3.0, C_3 = 1.1, n = 0.75$





- Goody's model produces the -5 slope seen at high frequencies and the ~-0.8 slope seen at mid frequencies
- Other models exist (e.g. due to Howe/Chase) that are algebraically simpler but less accurate (see section 12.1)
- A computational approach based on the Poisson equation (commonly called the TNO method) can also be used, see section 12.5

Pressure wavenumber frequency spectrum



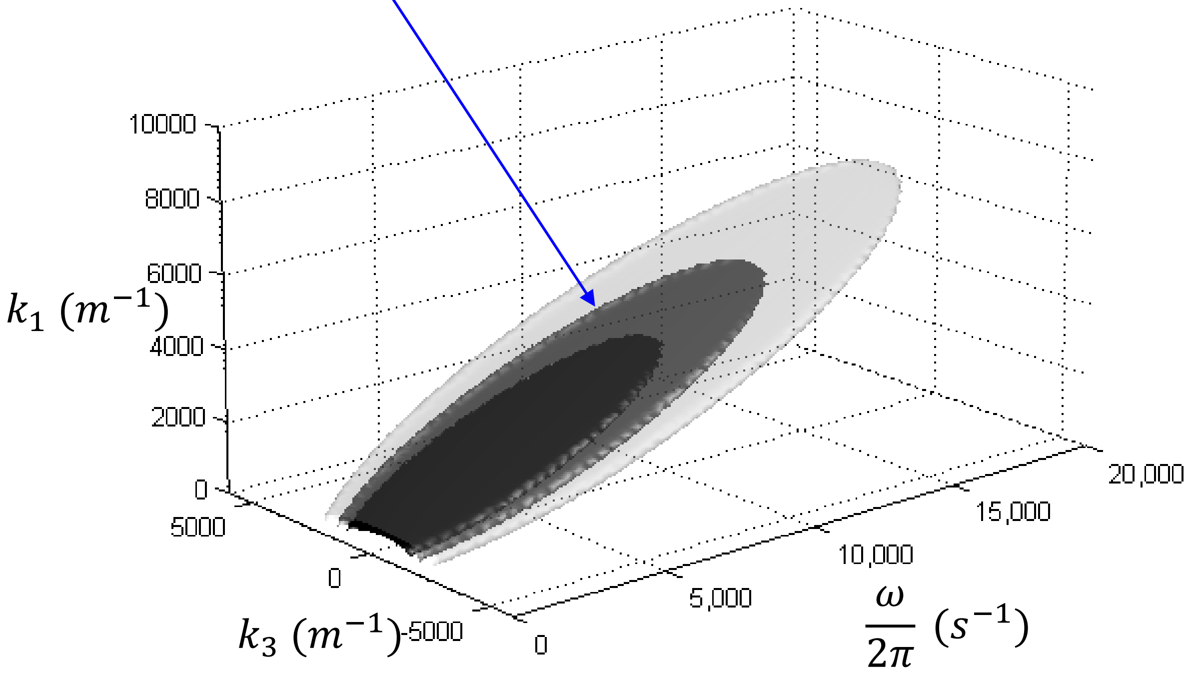
Corcos model

Eq. 12.2.1

$$\Phi_{pp}(k_1, k_3, \omega) = \frac{S_{pp}(\omega)U_c^2}{\pi^2\omega^2} \frac{a_1 a_3}{\left[a_1^2 + ((U_c k_1)/\omega - 1)^2\right]\left[a_3^2 + \left(U_c^2 k_3^2\right)/\omega^2\right]}$$

where U_c is the convection velocity, $S_{pp}(\omega)$ is the double-sided wall pressure frequency spectrum (e.g. from the Goody model), $a_1 = 0.1$, and $a_3 = 0.77$

Convective ridge

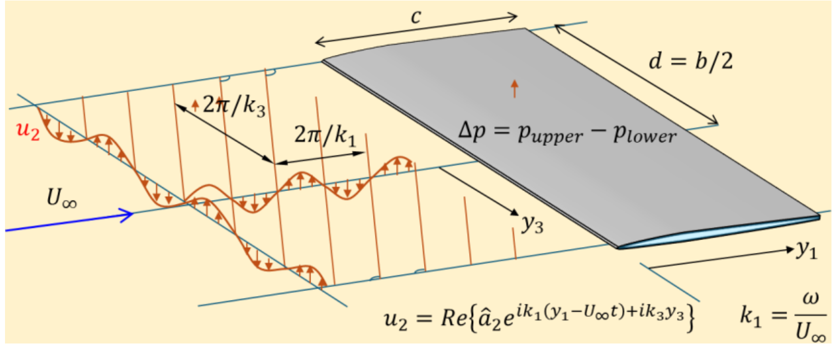


There is also the Chase model – regarded as much more accurate, but also more complex, see 12.2

Plot made with Chase model for TBL on previous slide

Some things to remember

- The need for two-point statistics to define aeroacoustics sources
- Auto/cross correlation functions and spectra, wavenumber spectra, integral time and length scales
- There are nice, closed form, analytical models for spatial correlations and wavenumber spectra of velocity fluctuations in homogeneous isotropic turbulence
- These also fit quite well to experimental data from inhomogeneous turbulent flows more typically found in applications
- Experimental data is available for the two-point statistics of turbulent wakes and boundary layers
- Analytic expressions exist for the boundary wall pressure frequency spectra, and wavenumber-frequency spectra



- The upwash wavenumber spectrum on a plane through the turbulence

$$\phi_{22}(k_1, k_3) = \frac{4}{9\pi}\frac{\overline{u^2}}{k_e^2}\frac{(k_1^2+k_3^2)/k_e^2}{\left[1+(k_1^2+k_3^2)/k_e^2\right]^{7/3}}$$